



innovating communications

## The Centre Tecnològic de Telecomunicacions de Catalunya

*A gateway to advanced communication technologies*

SPACE-TIME CODING

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## CODIGOS OSTBC

The coder matrix

$$\underline{\underline{B}} \cdot \underline{\underline{B}}^H = \underline{\underline{I}}_{n_T}$$

The Tx and Rx signals as well as the estimated symbol are:

$$\begin{aligned} & \underline{\underline{X}}_{T,n} \quad \underline{\underline{B}} \cdot s1(n) \\ & \underline{\underline{X}}_{R,n} \quad \underline{\underline{H}} \cdot \underline{\underline{X}}_{T,n} \quad \underline{\underline{W}}_n \\ & s\hat{1} \quad \text{Traza} \quad \underline{\underline{B}}^H \underline{\underline{H}}^H \cdot \underline{\underline{X}}_{R,n} \end{aligned}$$

For nt=2 there are

several possibilities with entries entailing no operation

1	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1
0	1	0	1	0	1	0	1
1	0	1	0	1	0	1	0

...

For two PAM symbols (real) and 2 antennas

$$\underline{\underline{X}}_T \quad \underline{\underline{B}}_1^H \cdot \underline{s}1 \quad \underline{\underline{B}}_2^H \cdot \underline{s}2$$

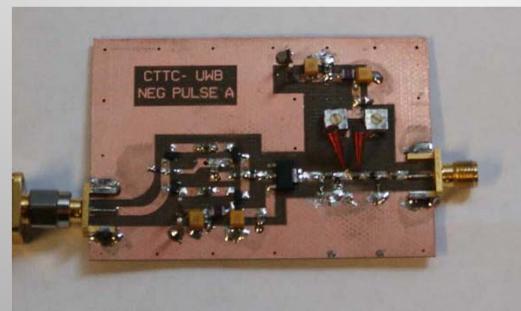
$$\widehat{s}1 \quad \text{Traza} \quad \underline{\underline{B}}_1^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{B}}_1 \cdot s1 \quad \text{Traza} \quad \underline{\underline{B}}_1^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{B}}_2 \cdot s2$$

*desired*    *ISI*

The no-ISI constrain is:

$$\text{Traza} \quad \underline{\underline{B}}_1^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{B}}_2 \quad \text{Traza} \quad \underline{\underline{R}}_H \cdot \underline{\underline{B}}_2 \cdot \underline{\underline{B}}_1^H \quad 0 \quad \underline{\underline{B}}_2 \cdot \underline{\underline{B}}_1^H \quad 0 ?$$

Not orthogonal just to be amicable



$$\underline{\underline{B}}_1 \cdot \underline{\underline{B}}_2^H \quad \underline{\underline{B}}_2 \cdot \underline{\underline{B}}_1^H$$



$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1.s1 \quad \underline{\underline{B}}_2.s2 \quad \begin{matrix} s1 & s2 \\ s2 & s1 \end{matrix}$$

To further achieve full-rate, we need two additional matrices, that being amicable in order to detect two imaginary parts, do not promote ISI with the real symbols.

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \underline{\underline{B}}_3 = \begin{pmatrix} B \\ B^H \end{pmatrix} \quad \underline{\underline{B}}_4 = \begin{pmatrix} B^H \\ B \end{pmatrix}$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1.s1 \quad \underline{\underline{B}}_2.s2 \quad j.\underline{\underline{B}}_3.s3 \quad j.\underline{\underline{B}}_4.s4$$

$$\underline{\underline{B}}_1.\underline{\underline{B}}_3^H \quad \underline{\underline{B}}_3.\underline{\underline{B}}_1^H$$

$$\underline{\underline{B}}_2.\underline{\underline{B}}_4^H \quad \underline{\underline{B}}_4.\underline{\underline{B}}_2^H$$

It is easy to check that  
this is the constraint---→



In summary:

$$\begin{array}{ccccc}
 \underline{\underline{B}}_1 \cdot \underline{\underline{B}}_1^H & \underline{\underline{B}}_2 \cdot \underline{\underline{B}}_2^H & \underline{\underline{B}}_3 \cdot \underline{\underline{B}}_3^H & \underline{\underline{B}}_4 \cdot \underline{\underline{B}}_4^H & \underline{\underline{I}}_2 \\
 \underline{\underline{B}}_1 \cdot \underline{\underline{B}}_2^H & \underline{\underline{B}}_2 \cdot \underline{\underline{B}}_1^H & \underline{\underline{B}}_4 \cdot \underline{\underline{B}}_3^H & \underline{\underline{B}}_3 \cdot \underline{\underline{B}}_4^H & \\
 \underline{\underline{B}}_1 \cdot \underline{\underline{B}}_3^H & \underline{\underline{B}}_3 \cdot \underline{\underline{B}}_1^H & \underline{\underline{B}}_2 \cdot \underline{\underline{B}}_4^H & \underline{\underline{B}}_4 \cdot \underline{\underline{B}}_2^H &
 \end{array}$$

The Alamouti's code:

$$\begin{array}{ccccccccc}
 \underline{\underline{B}}_1 & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \underline{\underline{B}}_2 & \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} & \underline{\underline{B}}_3 & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \underline{\underline{B}}_4 & \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \\
 X_T & \underline{\underline{B}}_1.s1 & \underline{\underline{B}}_2.s2 & \underline{\underline{B}}_3.j.s3 & \underline{\underline{B}}_4.j.s4 & & s1 & j.s3 & s2 & js4 & z1 & z2^* \\
 & & & & & s2 & j.s4 & s1 & j.s3 & z2 & z1^*
 \end{array}$$

The receiver----->

$$\widehat{s}1 \text{ Re } \text{Trazza } \underline{\underline{B}}_1^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{X}}_R$$

$$\widehat{s}2 \text{ Re } \text{Trazza } \underline{\underline{B}}_2^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{X}}_R$$

$$\widehat{s}3 \text{ Im } \text{Trazza } \underline{\underline{B}}_1^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{X}}_R$$

$$\widehat{s}4 \text{ Im } \text{Trazza } \underline{\underline{B}}_2^H \cdot \underline{\underline{R}}_H \cdot \underline{\underline{X}}_R$$



Unfortunately no such full-rate codes exist for any number of antennas. There are solution for rates lower than one like the code shown below for 4 antennas and rate  $\frac{3}{4}$ .

$$\begin{array}{cccc} s1 & 0 & s2 & s3 \\ 0 & s1 & s3^* & s2^* \\ s2^* & s3 & s1^* & 0 \\ s3^* & s2 & 0 & s1^* \end{array}$$



## ***Convolutional S-T Codes: Trellis codes***

output    input    state

$$\underline{x} \quad \underline{G}_1 \cdot \underline{a} \quad \underline{G}_2 \cdot \underline{b}$$

$\underline{a} = [a(1), a(2), \dots, a(R)]^T$  input bits

L bits per component output  $\underline{x}$

Measurement equation

$$\underline{b} \quad \underline{G}_3 \cdot \underline{a} \quad \underline{G}_4 \cdot \underline{b}$$

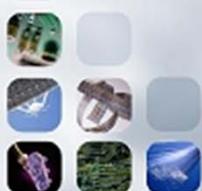
L=2 bits Components

0(00),1(01),2(10),3(11) that correspond to the four signals in a QPSK constellation

State equation

Code rate=R/L over  $n_T$  antennas

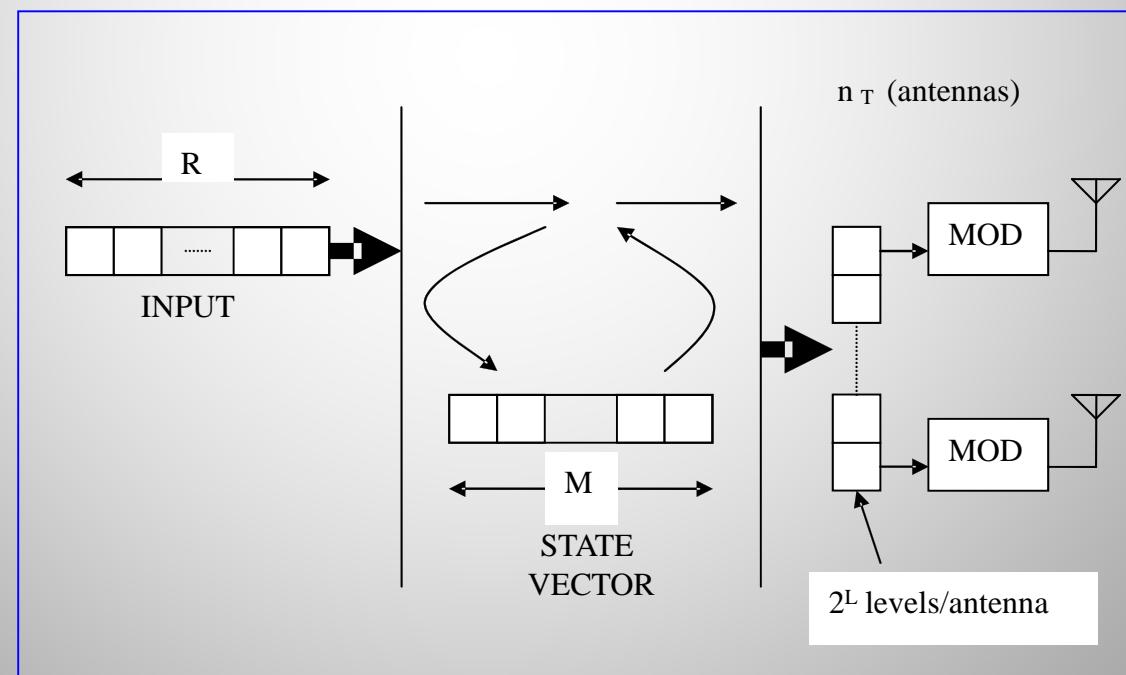
K bits for M components of the state vector  $\underline{b}$





## Code specification:

- Number of Tx antennas  $n_T$
- Bits/Hz or size of the radiated constellation  $L$
- Complexity at Tx or number of states  $2^{K \cdot M}$
- Code Rate equal to  $R/L$



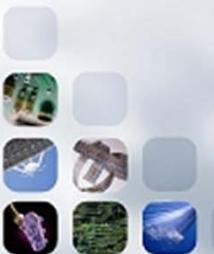


A more compact formulation of the state model is:

Grouping input and state vectors in a single one

$$\begin{array}{ccccccc} \underline{c} & a(1) & \dots & a(R) & b(1) & \dots & b(M) \\ \underline{x} & & & & \underline{\underline{G.c}} & & \end{array}$$

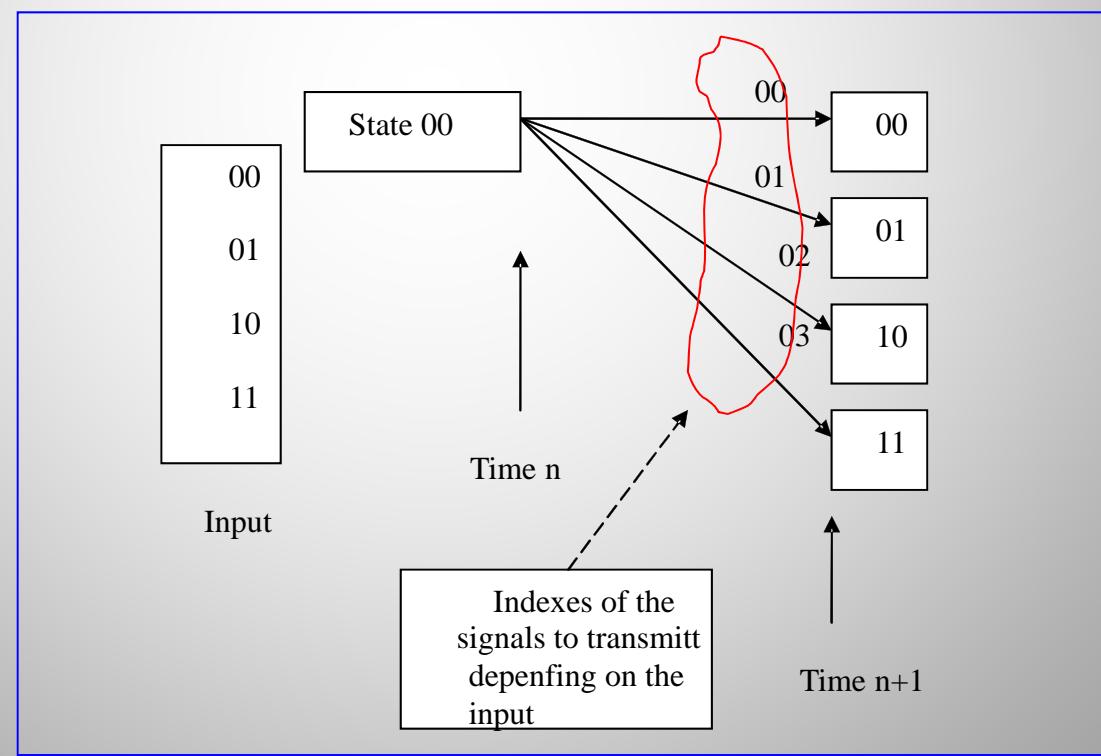
And, the state vector is formed by the last M bits after a shift of a given number of components vector  $\underline{c}$  from left to right

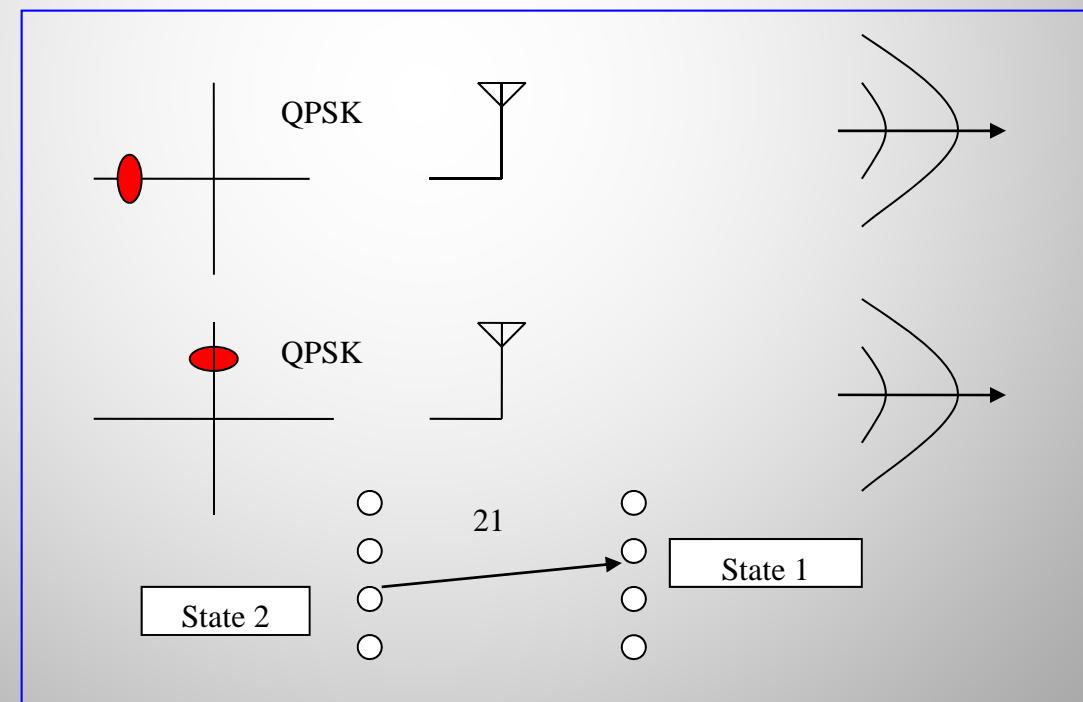
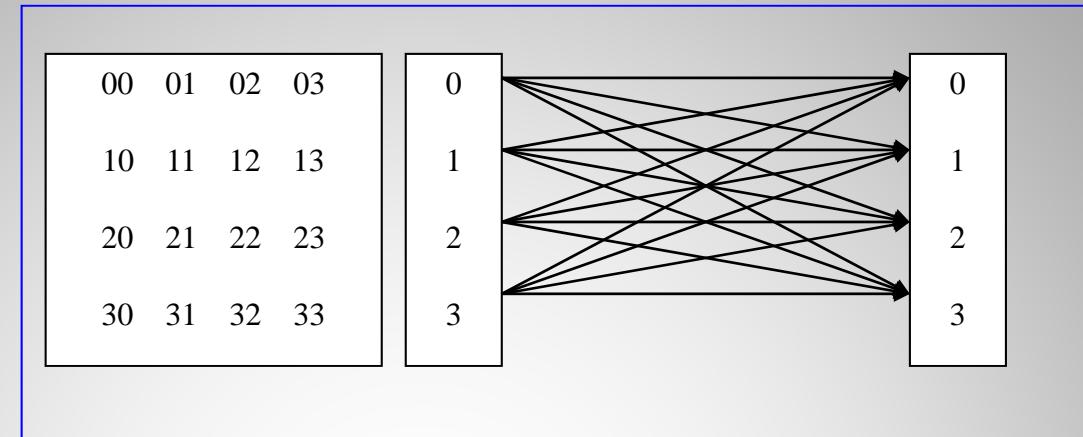


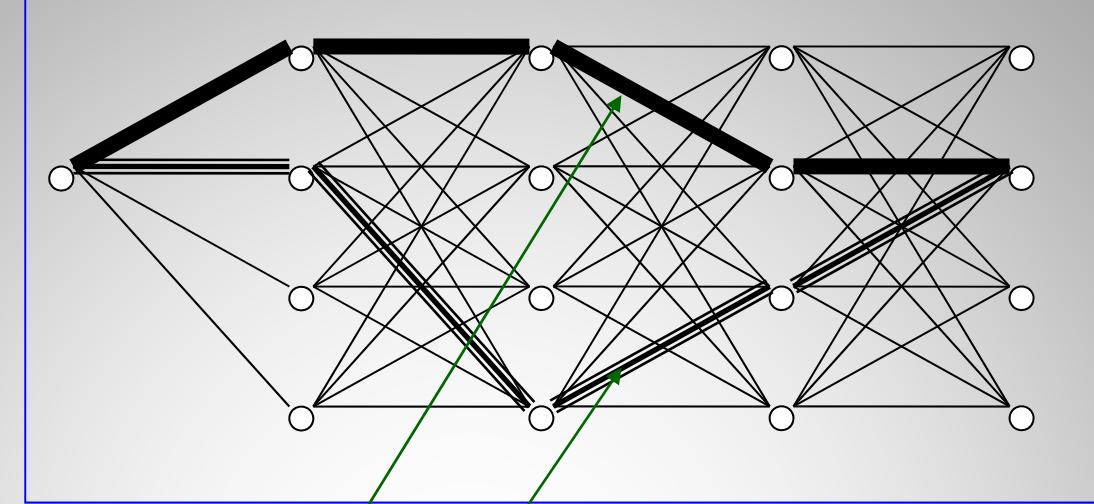
## *Code st2bh2est4rate1:*

$$a(1) \quad a(2) \quad b(1) \quad b(2) \cdot \begin{matrix} 0 & 2 \\ 0 & 1 \\ 2 & 0 \\ 1 & 0 \end{matrix} \quad x_1 \quad x_2$$

Modul 2  
operation for  
 $x_1$  and  $x_2$







For uniform code any two pair is good. Use the  $(0,0,0,0,0\dots)$  as reference.

Search for the lowest distance covered to recover the zero path  
(This will be the worst matrix A to be used in the BER upper bound)

$$\Pr \underline{s}_n | \underline{b}_n; n = 1, N \leq Q \sqrt{\frac{E_s}{2N_0} \cdot \text{Traza } \underline{\underline{R}}_H \cdot \underline{\underline{A}}}$$

For length of N channels access

$$\underline{A} = \sum_{n=1}^N \underline{s}_n \underline{b}_n \cdot \underline{s}_n \underline{b}_n^H$$

And the average BER

$$\Pr(\underline{I}_0 < \underline{I}_e) = k_1 \cdot \frac{1}{\det_{p=1}^{n_R} \underline{\underline{I}}_{=n_T} \frac{2E_s}{N_0} \cdot \underline{\underline{A}}_{=p}}$$

For moderate and high SNRs

$$k_1 \cdot \frac{2E_s}{N_0} \cdot \frac{1}{\det_{p=1}^{n_R \cdot n_T} \underline{\underline{A}}_{=p}}$$



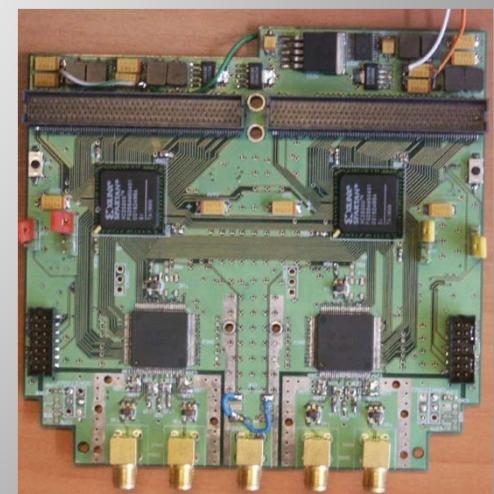
or

$$\Pr \underline{I}_0 = \underline{I}_e = k_1 \cdot \frac{2E_S}{N_0}^{n_R \cdot n_T} \cdot \frac{1}{\det_{p=1}^{\underline{A}_{\underline{A}=p}} \underline{A}} = k_2 \cdot \frac{2E_S}{N_0}^{n_R \cdot n_T} \cdot \det_{p=1}^{\underline{A}_{\underline{A}=p}} \underline{A}$$



*Ganancia*       $\det \underline{A}^r$

With  $r=1/n_T$



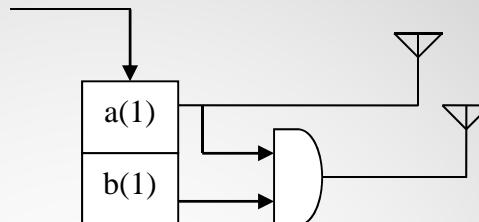
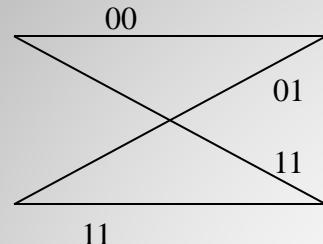
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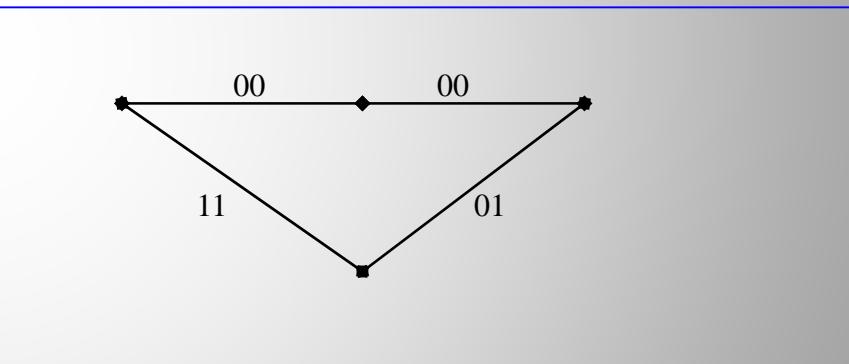
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de Catalunya

For st2bh1est2rate0.5

$$\underline{G} \begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix}$$



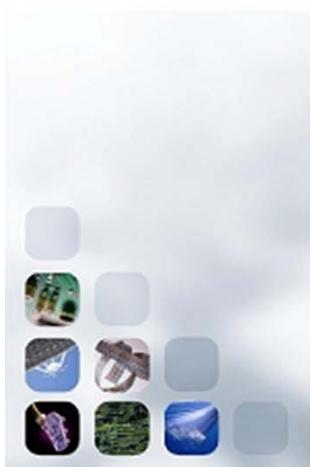
$$\underline{s}_1 \quad \underline{b}_1 \quad \begin{matrix} 2 \\ 2 \end{matrix} \quad y \quad \underline{s}_2 \quad \underline{b}_2 \quad \begin{matrix} 0 \\ 2 \end{matrix}$$



$$\underline{A} \begin{matrix} 2 & . & 2 & 2 \\ 2 & . & 0 & 2 \end{matrix} \begin{matrix} 0 & 4 & 4 & 0 & 0 & 4 & 4 \\ 4 & 4 & 0 & 4 & 4 & 8 \end{matrix}$$

$$\det \underline{A} = 16 \quad \text{Ganancia} = \sqrt{16} = 4$$





St2bh2est8	Gain $\sqrt{22}$	$\underline{\underline{G}}^T$	0 2 1 0 2 2 1 0 2 2
St2bh2est16	Gain $\sqrt{32}$	$\underline{\underline{G}}^T$	0 2 1 1 2 0 2 2 1 2 0 2
St2bh1est2	Gain 4	$\underline{\underline{G}}^T$	0 1 1 1
St2bh1est4	Gain $\sqrt{48}$	$\underline{\underline{G}}^T$	0 2 1 0 2 2 1 0 2 2
St2bh1est8	Gain $\sqrt{80}$	$\underline{\underline{G}}^T$	1 0 1 1 1 1 0 1
St2bh1est16	Gain $\sqrt{128}$	$\underline{\underline{G}}^T$	0 1 0 1 1 1 0 1 0 1 1 0 0 1
St3bh1est8	Gain $\sqrt[3]{256}$	$\underline{\underline{G}}^T$	1 0 1 0 1 1 1 1





## *Codes for No-CSI at Rx*

Assume that  $\log_2 M$  bits have to be transmitted using  $N$  access to the channel with  $n_T$  antennas. We will use  $M$  matrixes of  $n_T$  by  $N$

$$\underline{\underline{C}}_m \quad m = 1, M$$

$$\underline{\underline{C}}_m \cdot \underline{\underline{C}}_m^H \quad \underline{\underline{I}}_{n_T} \quad m = 1, M \quad \text{UPA at Tx}$$

H matrix is random-> Average BER

$$\text{At Rx} \rightarrow \underline{\underline{Y}}_R \quad \frac{2E_s}{N_0} \quad \cdot \underline{\underline{H}} \cdot \underline{\underline{C}}_0 \quad \underline{\underline{W}} \quad \frac{1}{\sqrt{2}} \cdot \underline{\underline{H}} \cdot \underline{\underline{C}}_0 \quad \underline{\underline{W}}$$

$$= E \underline{\underline{Y}}_R^H \underline{\underline{Y}}_R \stackrel{I}{=} \underline{\underline{C}}_0^H \cdot E \underline{\underline{H}}^H \underline{\underline{H}} \stackrel{I}{=} \underline{\underline{C}}_0 \stackrel{I}{=} \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{R}}_{HA} \cdot \underline{\underline{C}}_0 \stackrel{I}{=} \left| H_0 \right|^2 \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_0$$

The ML receiver is:

$$\Pr \underline{\underline{Y}}_R / \underline{\underline{C}}_0 = k_0 \cdot \exp \text{ Traza } \underline{\underline{Y}}_R^{-1} \underline{\underline{Y}}_R^H$$

$$k_0 \cdot \exp \text{ Traza } \underline{\underline{Y}}_R \stackrel{I}{=} - \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{Y}}_R^H$$

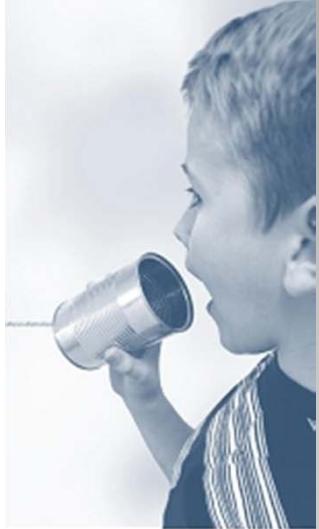
$$\hat{m} = \arg \max_{\underline{\underline{C}}_m, m=1,M} \text{ Traza } \underline{\underline{Y}}_R \cdot \underline{\underline{C}}_m^H \cdot \underline{\underline{C}}_m \cdot \underline{\underline{Y}}_R^H$$

Perfect detection occurs when:

$$\text{Traza } \underline{\underline{Y}}_R \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{Y}}_R^H \quad \text{Traza } \underline{\underline{Y}}_R \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{Y}}_R^H$$



$$Traza \underline{\underline{H}} \cdot \underline{\underline{I}} = \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{H}}^H = 2^{-1/2} \cdot \text{Re } Traza \underline{\underline{W}} \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{H}}^H$$



$$\Pr \underline{\underline{C}}_0 = \underline{\underline{C}}_1 = Q \sqrt{\frac{2E_s}{N_0}} \cdot Traza \underline{\underline{H}} \cdot \underline{\underline{A}}_{NOCSI} \cdot \underline{\underline{H}}^H$$

$$k_1 \cdot \exp \left( -\frac{2E_s}{N_0} \cdot Traza \underline{\underline{H}} \cdot \underline{\underline{A}}_{NOCSI} \cdot \underline{\underline{H}}^H \right)$$

$$\Pr^{AVE} k_2 = \frac{1}{\det \underline{\underline{I}} - \frac{E_s}{2 \cdot N_0} \cdot \underline{\underline{A}}_{NOCSI} \cdot \underline{\underline{I}}^H}$$

It is important to remark the loss due to the absence of CSI at Rx

$$E_S = \frac{N}{n_T} \cdot E_T$$

$$\begin{aligned} & \underline{\underline{A}}_{CSI} = \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_1^H \\ & \underline{\underline{A}}_{NOCSI} = \underline{\underline{I}} = \underline{\underline{C}}_0^H \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{C}}_0 \end{aligned}$$



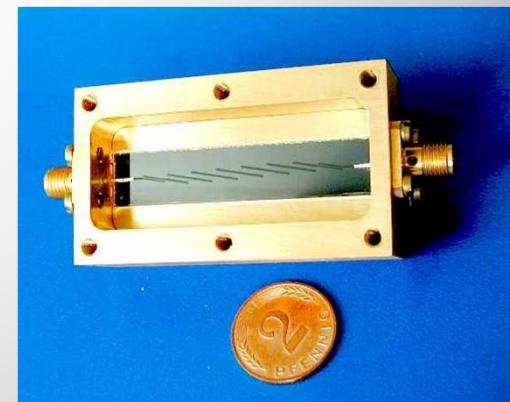
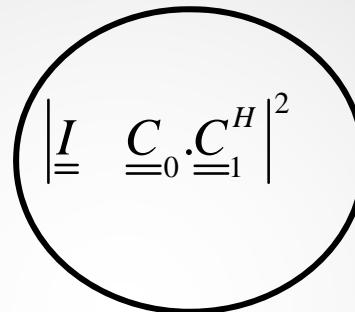
In order to compare both cases, note that:

$$\left| \underline{\underline{I}} - \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H \right|^2 = \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0^H = \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H = \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0^H$$

$$2 \cdot \underline{\underline{I}} = \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H = \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0^H = \underline{\underline{A}}_{NOCSI} = \underline{\underline{A}}_{CSI} = \underline{\underline{A}}_{NOCSI}$$

Así pues,

$$\underline{\underline{A}}_{CSI} = \underline{\underline{A}}_{NOCSI} = \left| \underline{\underline{I}} - \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H \right|^2$$





## Códigos ST Diferenciales

Assuming that there is CSi at Rx

$$\left| \underline{\underline{X}}_R - E_s^{1/2} \cdot \underline{\underline{H}} \cdot \underline{\underline{C}}_m \right|_F \leq \hat{C} = \max_{\underline{\underline{C}}_m; m=1,M} \text{Re Traza } \underline{\underline{H}} \cdot \underline{\underline{C}}_m \cdot \underline{\underline{X}}_R$$

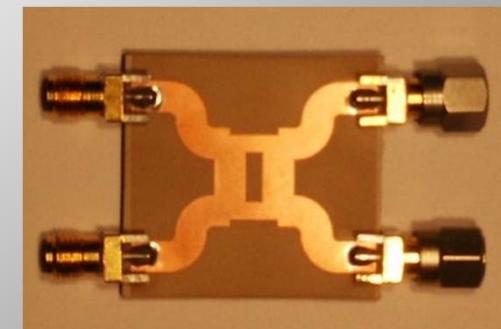
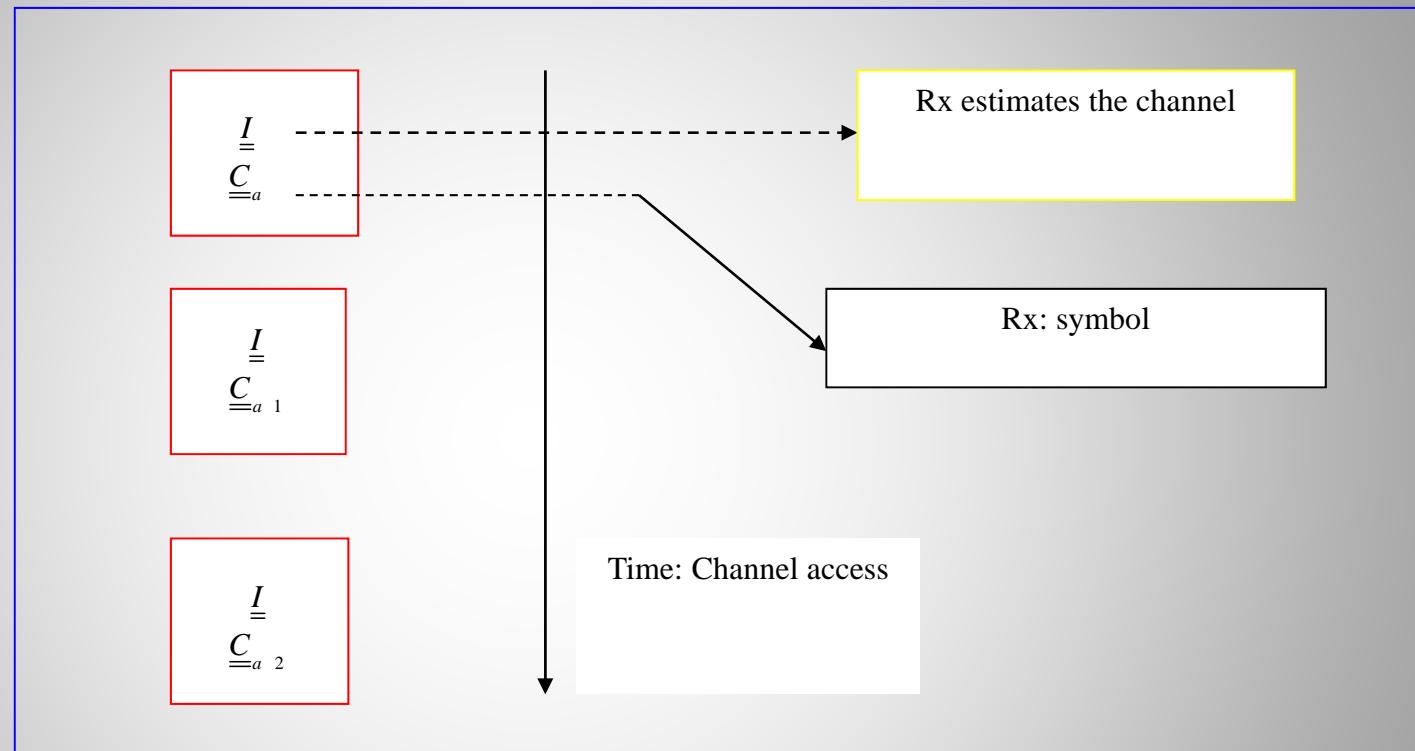
The probability of error is:

$$Pe = \Pr \left( \underline{\underline{C}}_m = \underline{\underline{C}}_n ; \tilde{\underline{\underline{C}}} = \underline{\underline{C}}_m \neq \underline{\underline{C}}_n \right) = k_0 \cdot Q \left( \sqrt{\frac{E_s}{2N_0}} \cdot \text{Traza } \underline{\underline{R}}_H \cdot \tilde{\underline{\underline{C}}} \cdot \tilde{\underline{\underline{C}}}^H \right)$$

$$Pe = k_1 \cdot \exp \left( - \frac{E_s}{4N_0} \cdot \text{Traza } \underline{\underline{R}}_H \cdot \tilde{\underline{\underline{C}}} \cdot \tilde{\underline{\underline{C}}}^H \right) = k_1 \cdot \exp \left( - \frac{E_s}{4N_0} \cdot \sum_{p=1}^{n_R} h_p^H \cdot \tilde{\underline{\underline{C}}} \cdot \tilde{\underline{\underline{C}}}^H \cdot h_p \right)$$

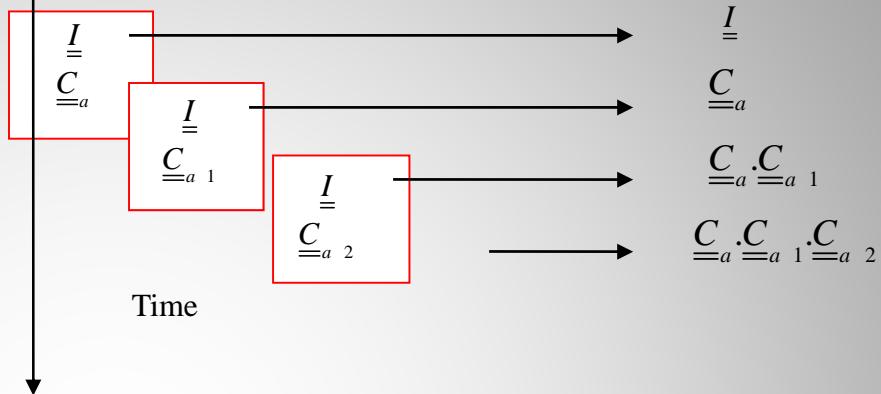
$$Pe^{aver} = k_2 \cdot \frac{1}{\det \underline{\underline{I}} - \frac{E_s}{4N_0} \cdot \tilde{\underline{\underline{C}}} \cdot \tilde{\underline{\underline{C}}}^H} \cdot \sum_{j=1}^{n_R}$$

*Using two symbols first estimate the channel  
second to decode.*





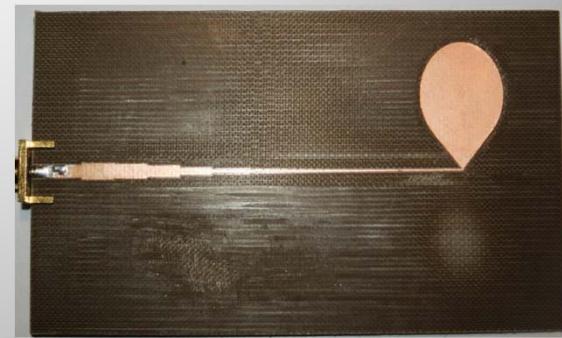
## The differential receiver



The desired word  
received is  $Z_{n-1} \dots a_1 \underline{C}_{n-a}$

And the received  
snapshot

$$\underline{X}_{R,n-1} \quad \underline{H} \cdot \underline{Z}_{n-1} \quad \underline{W}_{n-1}$$





Hereafter it is shown that the ST code produces, in fact, a new channel together with 3 dB. increase of noise.

$$\begin{array}{cccccc} \underline{\underline{X}}_{R,n} & \underline{\underline{H}} \cdot \underline{\underline{Z}}_{n-1} \cdot \underline{\underline{C}}_n & \underline{\underline{W}}_n & \underline{\underline{X}}_{R,n-1} & \underline{\underline{W}}_{n-1} \cdot \underline{\underline{C}}_n & \underline{\underline{W}}_n \\ \underline{\underline{X}}_{R,n} & \underline{\underline{X}}_{R,n-1} \cdot \underline{\underline{C}}_n & \underline{\underline{W}}_n & \underline{\underline{W}}_{n-1} \cdot \underline{\underline{C}}_n & \underline{\underline{H}}_{\text{nuevo}} \cdot \underline{\underline{C}}_n & \underline{\underline{W}}_{\text{nuevo}} \end{array}$$

Regardless the system is full-rate the decoder requires of two received codewords

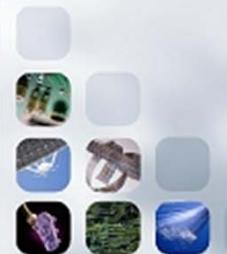
$$\left| \underline{\underline{X}}_{R,n} \quad \underline{\underline{X}}_{R,n-1} \cdot \underline{\underline{C}}_n \right|_F \quad \hat{\underline{\underline{C}}} = \underset{\underline{\underline{C}}_n; n=1, M}{\text{Max}} \quad \text{Re } \text{Traz} \underline{\underline{X}}_{R,n-1} \cdot \underline{\underline{C}}_n \cdot \underline{\underline{X}}_{R,n}^H$$

Also, the matrix A for the average BER is as follows:

$$\begin{aligned}
 2 \underline{\underline{I}} = & \frac{\underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{Z}}_{k-1}^H}{\underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0^H \cdot \underline{\underline{Z}}_{k-1}^H} \\
 & \cdot \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_1 + \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{Z}}_{k-1}^H \\
 & \cdot \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1 \cdot \underline{\underline{C}}_0 \cdot \underline{\underline{C}}_1^H \cdot \underline{\underline{Z}}_{k-1}^H = \underline{\underline{A}}_{DIF}^H
 \end{aligned}$$

Where, taking into account the orthogonal character of the received codewords and the commutative property of the determinant, results identical to the CSI at Rx case with 3dB loss.





## *Some examples of differential ST codes*

For 1 bit rate only two matrixes

$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

With initial  $D \equiv \begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$

BPSK as constellation and rate 0.25 and 2 antennas, Gain 4

2 bits, 2 antennas, rate 0.5 → four matrixes,  
BPSK, Gain 4

$$\begin{matrix} 1 & 0 & 0 & 1 \\ 0 & 1 & ' & 1 & 0 \end{matrix}$$



QPSK, 2bits/seg/Hz, 8 codewords, Code gain4,  
Rate 1

$$\begin{matrix} 1 & 0 & j & 0 & 0 & 1 & 0 & j \\ 0 & 1 & ' & 0 & j & ' & 1 & 0 & ' & j & 0 \end{matrix}$$

“quaternion” similar to Alamouti’s code

For higher rates the codewords are formed as:

$$w_Q = \exp(j2\pi/Q)$$

$$\begin{matrix} 0 & w_Q^m & ; m & 0, & Q & 1 \\ 1 & 0 \end{matrix}$$

Q=8 Rate 2 Gain 1.531 // Q=16 Rate 2.5 Gain 0.7804



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innovating communications

## The Centre Tecnològic de Telecomunicacions de Catalunya

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## *A compact formulation for two channel access:*

Two data received

$$\underline{\underline{X}}_{R,k-1} \quad \underline{\underline{X}}_{R,k}$$

New codeword

$$\underline{\underline{Z}}_{k-1} \quad \underline{\underline{Z}}_{k-1} \cdot \underline{\underline{C}}_k \quad \overline{\underline{\underline{C}}}_k$$

with

$$\begin{aligned} \overline{\underline{\underline{C}}}^H \cdot \overline{\underline{\underline{C}}} &= \frac{\underline{\underline{I}}}{\underline{\underline{C}}^H} \quad \frac{\underline{\underline{C}}_k}{\underline{\underline{I}}} \\ &\quad y \quad \overline{\underline{\underline{C}}} \cdot \overline{\underline{\underline{C}}}^H = 2 \cdot \underline{\underline{I}} \end{aligned}$$

Optimum detector for no-CSI at Rx that arrives to the same result

$$\begin{aligned} \text{Traza } \underline{\underline{X}}_{R,k-1} \cdot \underline{\underline{X}}_{R,k} &= \frac{\underline{\underline{I}}}{\underline{\underline{C}}^H} \quad \frac{\underline{\underline{C}}_k}{\underline{\underline{I}}} \quad \frac{\underline{\underline{X}}_{R,k-1}^H}{\underline{\underline{X}}_{R,k}} \\ &= \frac{\underline{\underline{I}}}{\underline{\underline{I}}} \quad \frac{\underline{\underline{X}}_{R,k}}{\underline{\underline{X}}_{R,k}} \end{aligned}$$

$$\text{Traza } \underline{\underline{X}}_{R,k} \underline{\underline{C}}^H \underline{\underline{X}}_{R,k-1}^H$$

