

innovating communications

The Centre Tecnològic de Telecomunicacions de Catalunya

A gateway to advanced communication technologies

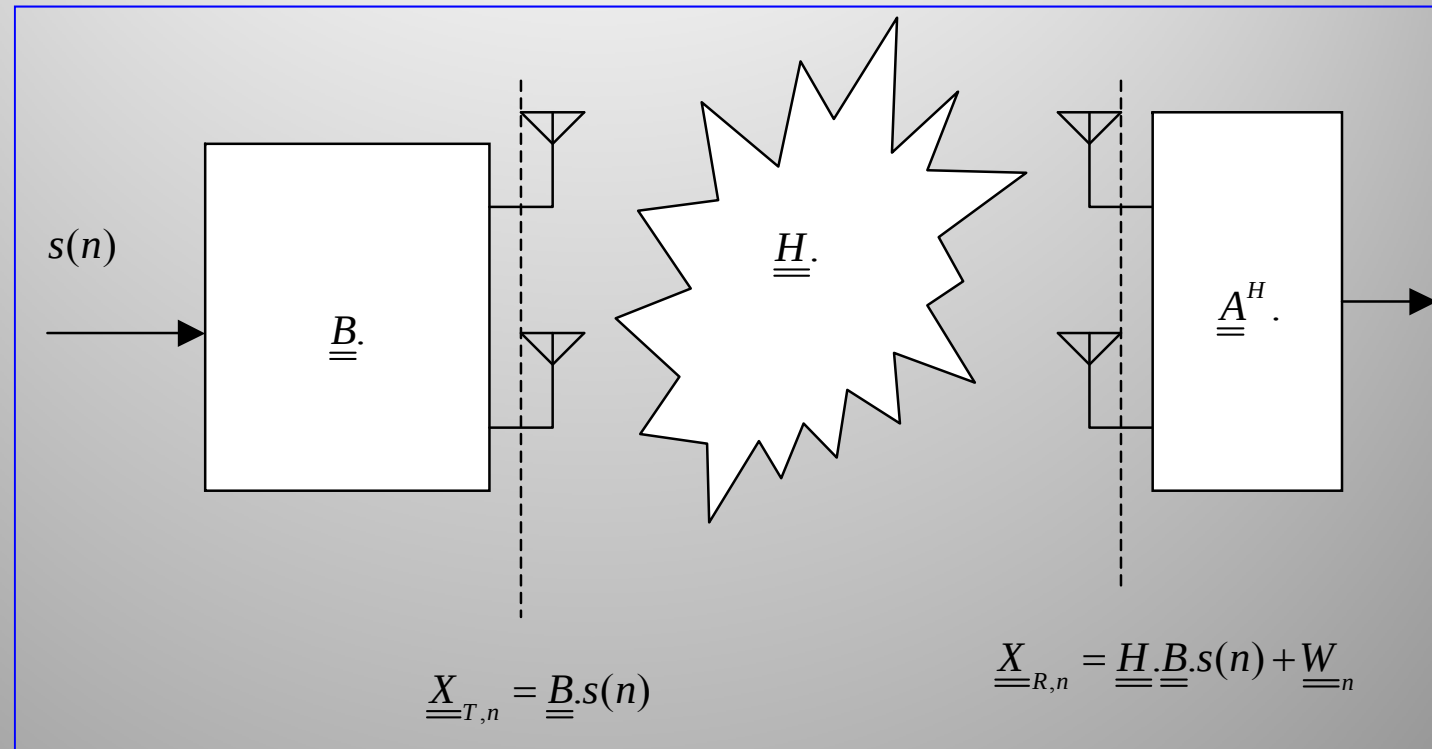
MIMO1: Single symbol over flat fading CSIT/CSIR

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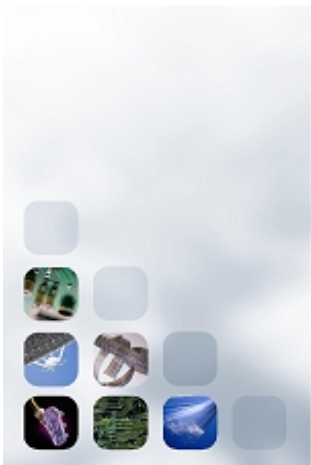
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The Scenario



Flat Fading, CSIT and CSIR, Single Symbol



The ML Receiver

$$\underline{\underline{X}}_{T,n} = \underline{\underline{B}}.s(n)$$

$$E_T = \text{trace}(\underline{\underline{B}}.\underline{\underline{B}}^H) E(|s(n)|^2) = d^2 \cdot \frac{2^{n_s} - 1}{6} \cdot \text{trace}(\underline{\underline{B}}.\underline{\underline{B}}^H)$$

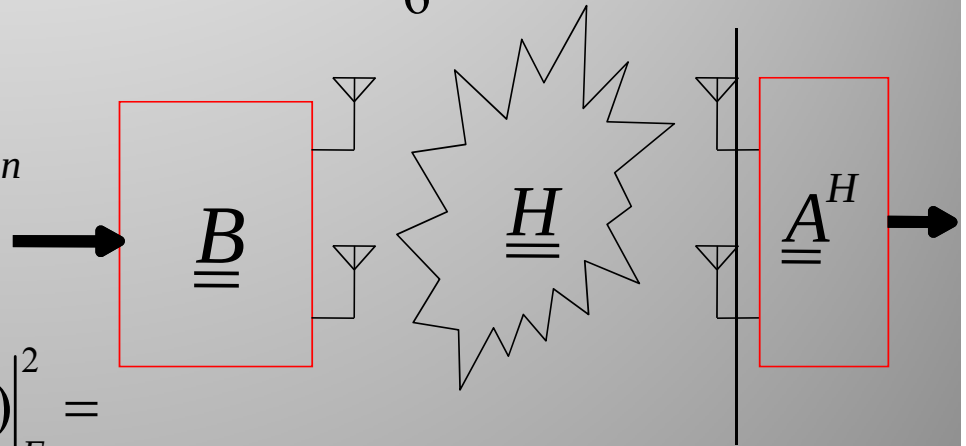
$$\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{B}}.s(n) + \underline{\underline{W}}_n$$

The likelihood

$$\Lambda(s(n)) = -\left| \underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n) \right|_F^2 =$$

$$= -\text{Traza} \left[\left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n) \right) \cdot \left(\underline{\underline{X}}_{R,n} - \underline{\underline{H}}.\underline{\underline{B}}.s(n) \right)^H \right] =$$

$$\rightarrow 2.\text{Re} \left[s(n)^* \cdot \text{Traza} \left(\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{X}}_{R,n} \right) \right] - |s(n)|^2 \cdot \text{Traza} \left[\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \right]$$



ML Receiver

$$\underline{\underline{A}} = \underline{\underline{H}} \cdot \underline{\underline{B}}$$

Optimum receiver

when $\Lambda(s(n)) > \Lambda(s(m)) \quad \forall m \neq n$ decide $s(n)$

After some manipulations and with $\tilde{s}(n) = s(n) - s(m)$

$$\text{Traza} \left[\left(\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \right) \cdot |\tilde{s}(n)|^2 \right] > 2 \cdot \text{Re} \left[\tilde{s}(n) \cdot \text{Traza} \left(\underline{\underline{W}}_n^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \right) \right]$$

The global Tx-H matrix

define

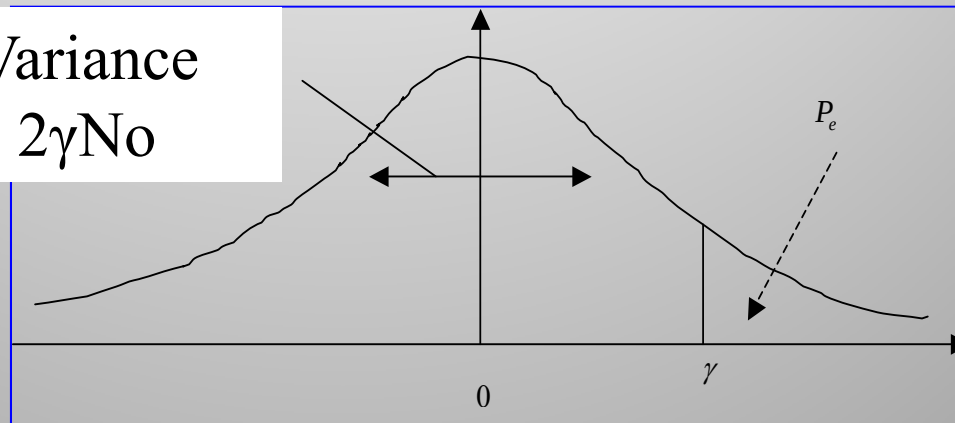
$$\gamma = d^2 \cdot \text{Trace} \left(\underline{\underline{H}}^H \cdot \underline{\underline{R}}_0^{-1} \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) = d^2 \cdot \text{Trace} \left[\underline{\underline{R}}_H \cdot \left(\underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) \right]$$



The Probability of Error

$$\text{Traza} \left[\left(\underline{\underline{B}}^H \cdot \underline{\underline{H}}^H \cdot \underline{\underline{H}} \cdot \underline{\underline{B}} \right) \cdot |\tilde{s}(n)|^2 \right] > 2 \cdot \text{Re} \left[\tilde{s}(n) \cdot \text{Traza} \left(\underline{\underline{W}}_n^H \underline{\underline{H}} \cdot \underline{\underline{B}} \right) \right]$$

Variance
 $2\gamma N_0$



Error only to the nearest neighborhood

$$\Pr(s(n) \rightarrow s(m)) = P_e = Q \left(\sqrt{\frac{\gamma}{2 \cdot N_0}} \right)$$





Tx Optimization

$$\begin{aligned}
 & d^2 \text{trace}(\underline{\underline{R}}_H \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H) \Big|_{MAX} \\
 & s.t. \quad E_s \cdot \text{trace}(\underline{\underline{B}} \cdot \underline{\underline{B}}^H) = E_T
 \end{aligned}$$

Max. Quality with a given power budget

The channel matrix

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H \underline{\underline{R}}_0^{-1} \underline{\underline{H}}$$

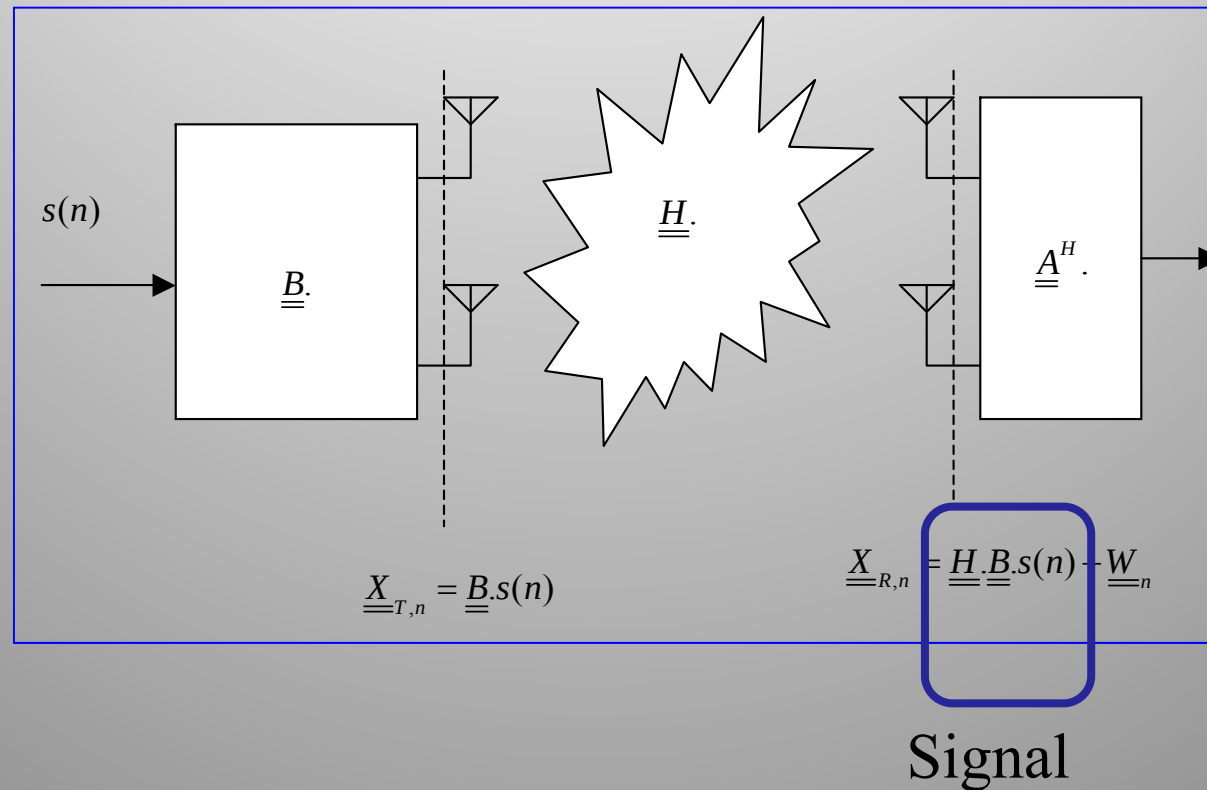
The Tx matrix and constraint

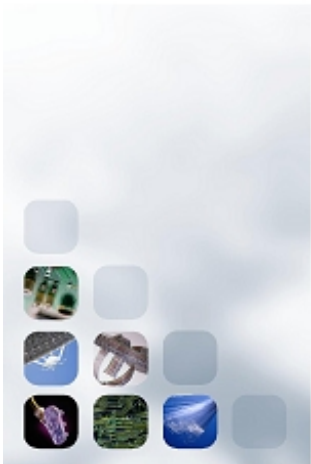
$$\underline{\underline{Q}} = (\underline{\underline{B}} \cdot \underline{\underline{B}}^H) \cdot E_s = (\underline{\underline{B}} \cdot \underline{\underline{B}}^H) \cdot d^2 \cdot \frac{2^{n_b} - 1}{6}$$

$$\text{Trace}(\underline{\underline{Q}}) = d^2 \cdot \frac{2^{n_b} - 1}{6} \cdot \text{Trace}(\underline{\underline{B}} \cdot \underline{\underline{B}}^H) \leq E_T$$

Maximum SNR Design (ML)

An alternative design (easy to perform) is to compute the SNR at the receiver output and then maximise it for the receiver and the transmitter.





At the receiver the estimated symbol will be:

$$\hat{s}(n) = \text{trace}(\underline{\underline{A}}^H \underline{\underline{H}} \underline{\underline{B}}) s(n) + \text{trace}(\underline{\underline{A}}^H \underline{\underline{W}}_n)$$

In consequence, the SNR will be:

$$SNR = \frac{\text{tr}^2(\underline{\underline{A}}^H \underline{\underline{H}} \underline{\underline{B}}) E_s}{\text{tr}(\underline{\underline{A}}^H \underline{\underline{R}}_0 \underline{\underline{A}})}$$

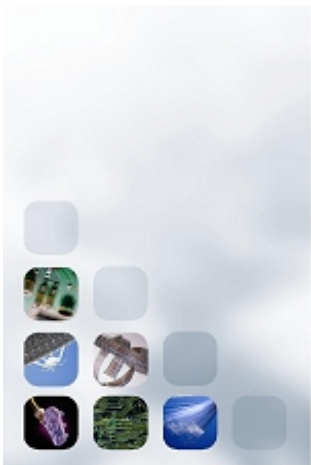
Since the trace is a linear operator
we use the SW un-equality as:

$$\text{tr}(\underline{\underline{u}}^H \underline{\underline{u}}) \text{tr}(\underline{\underline{v}}^H \underline{\underline{v}}) \leq \text{tr}^2(\underline{\underline{u}}^H \underline{\underline{v}})$$

with

$$\underline{\underline{u}} = \underline{\underline{R}}_0^{1/2} \underline{\underline{A}}$$

$$\underline{\underline{v}} = \underline{\underline{R}}_0^{-1/2} \underline{\underline{H}} \underline{\underline{B}}$$



$$SNR = \frac{tr^2(\underline{\underline{A}}^H \underline{\underline{H}} \underline{\underline{B}}) E_s}{tr(\underline{\underline{A}}^H \underline{\underline{R}}_0 \underline{\underline{A}})} = \frac{tr^2(\underline{\underline{u}}^H \underline{\underline{v}}) E_s}{tr(\underline{\underline{u}}^H \underline{\underline{u}})} \leq E_s tr(\underline{\underline{v}}^H \underline{\underline{v}})$$

Thus the maximum SNR is:

$$SNR = E_s tr(\underline{\underline{B}}^H [\underline{\underline{H}}^H \underline{\underline{R}}_0^{-1} \underline{\underline{H}}] \underline{\underline{B}}) = E_s tr(\underline{\underline{B}}^H \underline{\underline{R}}_C \underline{\underline{B}})$$

This maximum is achieved
when both matrixes are
proportional, then.....

The channel matrix
(gain/noise)

$$\underline{\underline{R}}_0^{1/2} \underline{\underline{A}} \propto \underline{\underline{R}}_0^{-1/2} \underline{\underline{H}} \underline{\underline{B}} \Rightarrow \underline{\underline{A}} = \underline{\underline{R}}_0^{-1} \underline{\underline{H}} \underline{\underline{B}}$$



Next, the SNR have to be maximized
with respect the transmitter matrix $\underline{\underline{B}}$

$$SNR = E_s \text{tr} \left(\underline{\underline{B}}^H \underline{\underline{R}}_C \underline{\underline{B}} \right)$$

Subject to the Tx power
constraint

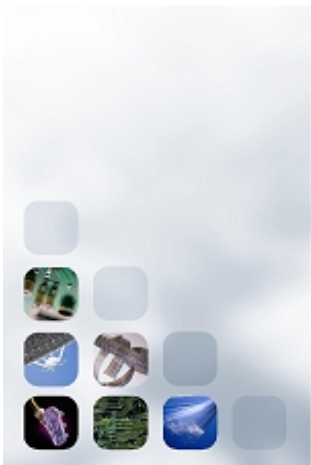
$$s.t. \quad E_s \cdot \text{trace} \left(\underline{\underline{B}} \underline{\underline{B}}^H \right) = E_T$$

To solve this problem we will use the following property:

$$\text{trace} \left(\underline{\underline{R}}_C \underline{\underline{B}} \underline{\underline{B}}^H \right) \leq \lambda_{\max} \left(\underline{\underline{R}}_C \right) \text{tr} \left(\underline{\underline{B}} \underline{\underline{B}}^H \right) = \lambda_{\max} \left(\underline{\underline{R}}_C \right) \frac{E_T}{E_s}$$

In summary, the
maximum SNR is....

$$SNR = E_T \lambda_{\max} \left(\underline{\underline{R}}_C \right)$$



Full CSIT Tx-precoder Design:

$$\begin{aligned} & d^2 \text{trace}(\underline{\underline{R}}_H \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H) \Big|_{MAX} \\ \text{s.t. } & E_s \cdot \text{trace}(\underline{\underline{B}} \cdot \underline{\underline{B}}^H) = E_T \end{aligned}$$

Since, for definite positive matrixes, we have:

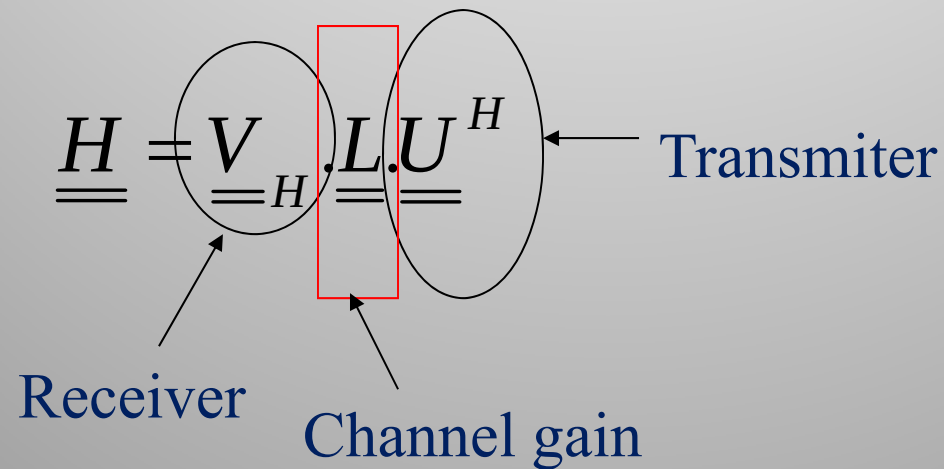
$$\text{Trace}(\underline{\underline{F}} \cdot \underline{\underline{G}}) \leq \lambda_{\max}(\underline{\underline{F}}) \cdot \text{Trace}(\underline{\underline{G}})$$

Equal when:

$$\underline{\underline{B}} = E_T \cdot \underline{\underline{e}}_{\max} \cdot \underline{\underline{cs}}^H \quad \forall \underline{\underline{cs}} \text{ unit norm}$$

Full CSIT Architecture and Performance

$$P_e = Q\left(\sqrt{\frac{E_T}{N_0} \cdot \lambda_{MAX}(\underline{\underline{R}}_H) \cdot \frac{3}{2^{n_s} - 1}}\right)$$



$$\underline{\underline{A}} = \underline{\underline{H}} \cdot \underline{\underline{B}} = \left(\sum_{r=1}^{\min(n_T, n_R)} \lambda_r \cdot \underline{\underline{v}}_r \cdot \underline{\underline{u}}_r^H \right) \cdot E_T \cdot \underline{\underline{u}}_{\max} = \lambda_{\max} \cdot E_T \cdot \underline{\underline{v}}_{\max}$$

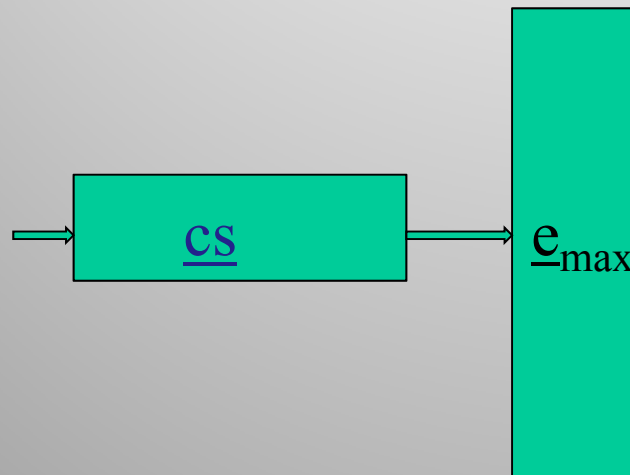




The maximum of the SNR results when

Where $\underline{e}_{\max}$ is the maximum eigenvector of the channel matrix

$$\underline{\underline{B}} = \underline{e}_{\max} \underline{CS}^H$$



This is a spreading vector which allows multiple access to the receiver. It has to be norm one in order to do not alter the transmitted power.

Note that for maximum SNR the space time processing at Tx reduces to a single beamformer (sometimes called beamforming solution)



Note that

$$\underline{\underline{R}}_C \underline{\underline{E}} = \underline{\underline{D}} \underline{\underline{E}}$$

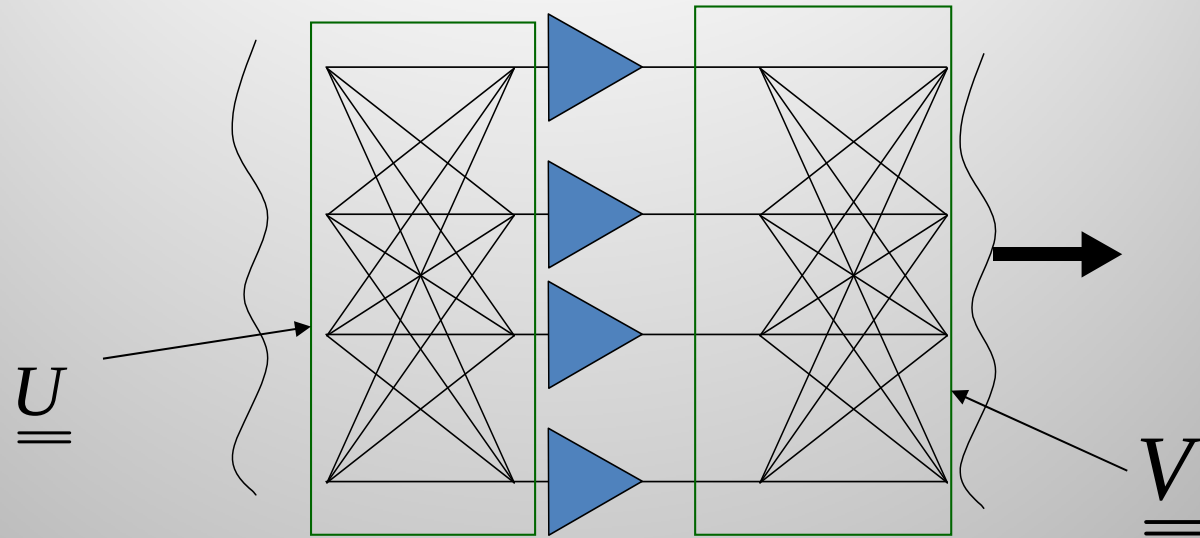
$$\underline{\underline{R}}_C = \underline{\underline{E}} \underline{\underline{D}} \underline{\underline{E}}^H = \sum_{q=1}^{n_0} \lambda_q \underline{\underline{e}}_q \underline{\underline{e}}_q^H$$

Or, using the svd of the equivalent channel

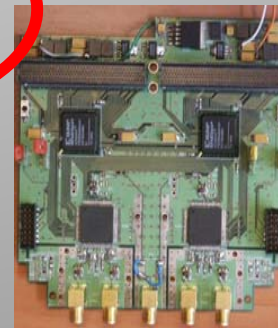
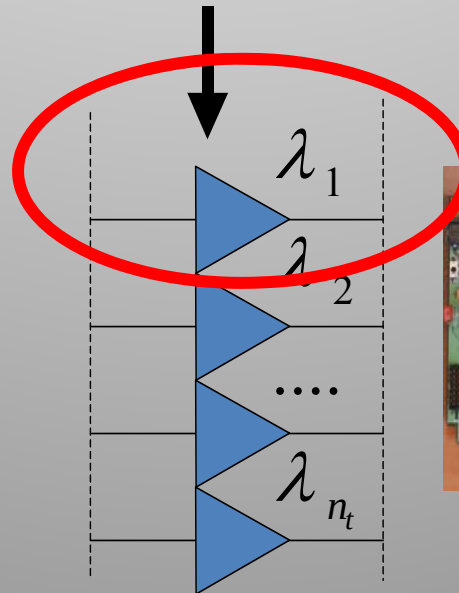
$$\underline{\underline{R}}_0^{-1/2} \underline{\underline{H}} = \underline{\underline{V}} \underline{\underline{D}}^{1/2} \underline{\underline{U}}^H \Rightarrow \begin{cases} \underline{\underline{B}} = \underline{\underline{U}} \\ \underline{\underline{A}} = \underline{\underline{R}}_0^{-1/2} \underline{\underline{V}} \end{cases}$$

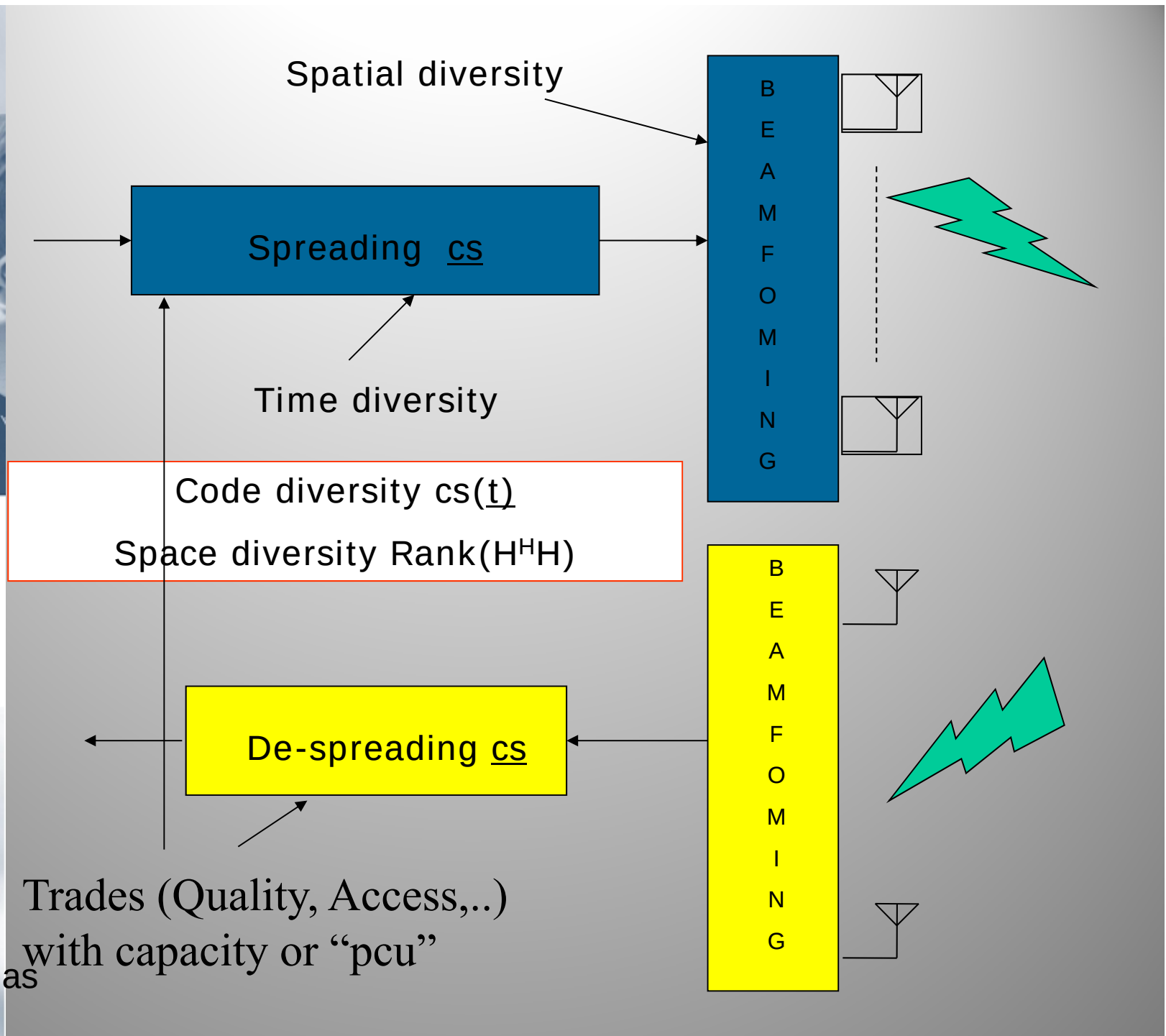
This is equivalent to say that B and A diagonalize the channel setting their values to U and V respectively. Each resulting channel, in number equal to the rank of the channel matrix, has a gain equal to the eigenvalue. THEN; THE BEST POLICY IS TO SEND ALL THE AVAILABLE POWER THROUGH THE BEST CHANNEL

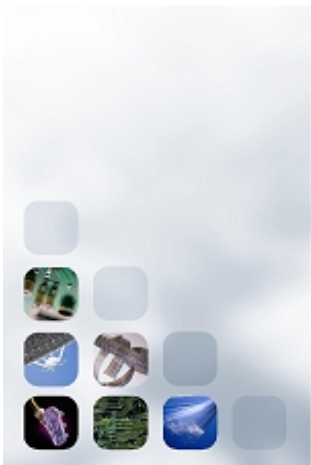
Intuitive Explanation



Just use the
best channel
for all the
available Tx
power







CSIR Only (Define a Game)

Define PLAYERS

TX Engineer

Channel

Define payoffs

Minimize P_E

Destroy
Rx Energy

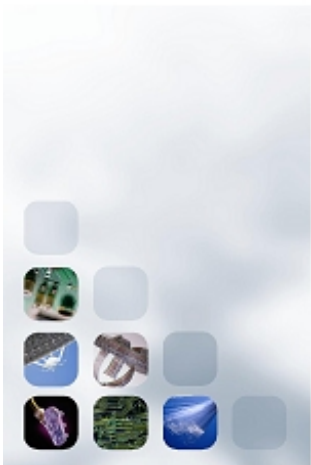
Define strategy of
the game

Channel plays first, Tx
engineer plays second. Only
one pass

Constrains/player

E_T

$\text{Trace}(\underline{R}_H) \geq \rho$



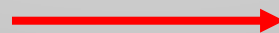
CSIR Only

$$\max_{\underline{\underline{Q}}} \left(\min_{\underline{\underline{R}}_h} \left(\underline{\underline{R}}_H \cdot \underline{\underline{Q}} \right) \right)$$

One play game

$$\underline{\underline{R}}_H = \underline{\underline{H}}^H \cdot \underline{\underline{R}}_0^{-1} \cdot \underline{\underline{H}} \quad \underline{\underline{Q}} = \underline{\underline{B}} \cdot \underline{\underline{B}}^H$$

$$\text{Traza} \left(\underline{\underline{R}}_H \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) \geq \lambda_{\min} \left(\underline{\underline{B}} \cdot \underline{\underline{B}}^H \right) \cdot \text{Traza} \left(\underline{\underline{R}}_H \right)$$



Channel plays

Tx Eng. plays as maximize the minimum eigenvalue when the trace is constrained

$$\underline{\underline{B}} = \left(\frac{E_T}{n_T} \right)^{1/2} \cdot \underline{\underline{\Pi}} \quad \text{with} \quad \underline{\underline{\Pi}} \cdot \underline{\underline{\Pi}}^H = \underline{\underline{I}}_{n_T}$$

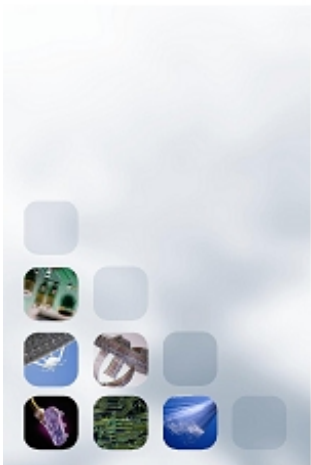


The CSIR Performance

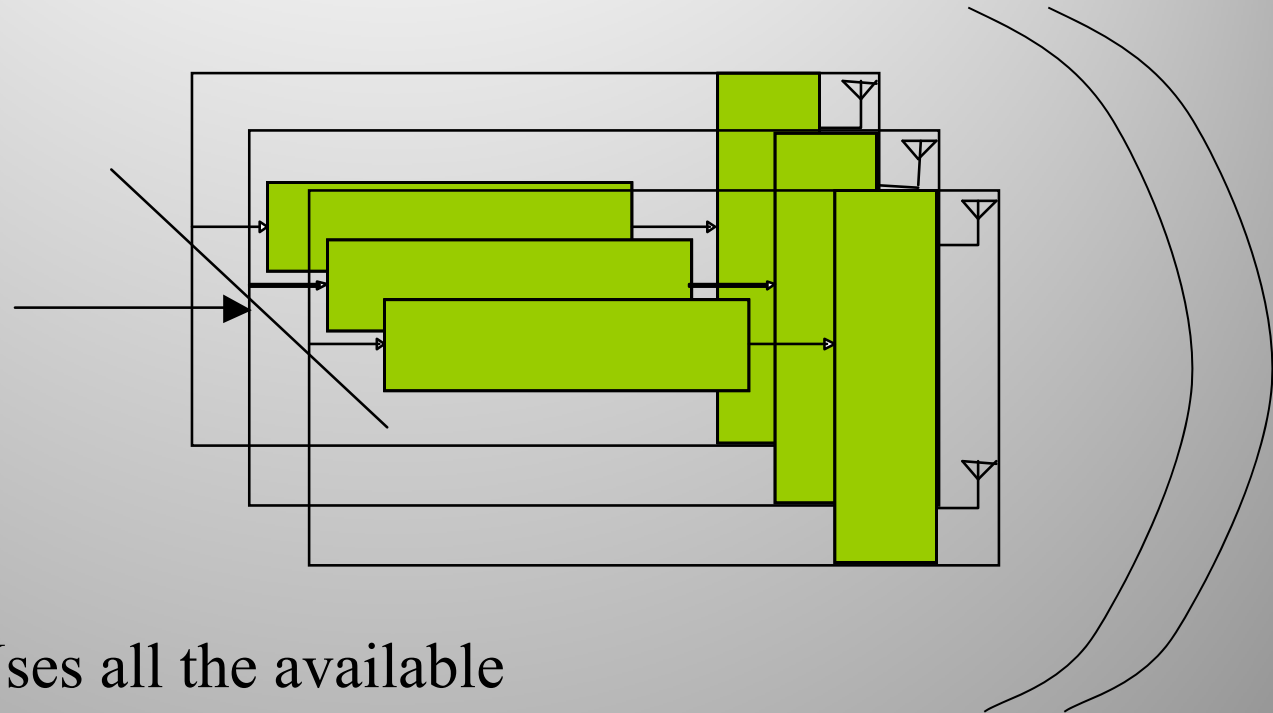
$$\underline{\underline{B}} = \underline{\underline{\Pi}} \cdot \sqrt{\frac{k_2}{n_T}} \quad y \quad \gamma = 4E_s \cdot \text{Traza}(\underline{\underline{R}}_H \cdot \underline{\underline{B}} \cdot \underline{\underline{B}}^H) = \frac{4 \cdot E_s \cdot k_2}{n_T} \cdot \text{Traza}(\underline{\underline{R}}_H)$$

$$P_e^{NO-CSI} = Q \left(\sqrt{\frac{E_T}{N_0} \cdot \left(\frac{\text{Traza}(\underline{\underline{R}}_H)}{n_T} \right) \cdot \left(\frac{3}{(2^{n_s} - 1)} \right)} \right)$$

We pass from the maximum eigenvalue gain to the arithmetic mean of the eigenvalues.



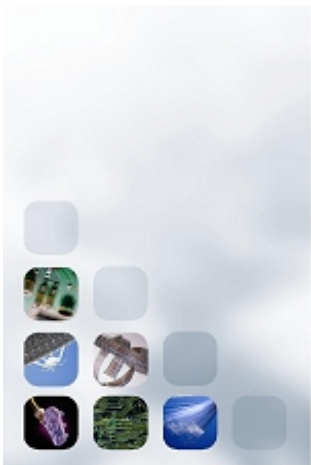
The CSIR only Strategy



Uses all the available
eigenmodes

UPA

Uniform Power
Allocation



PCU and Increase Rate

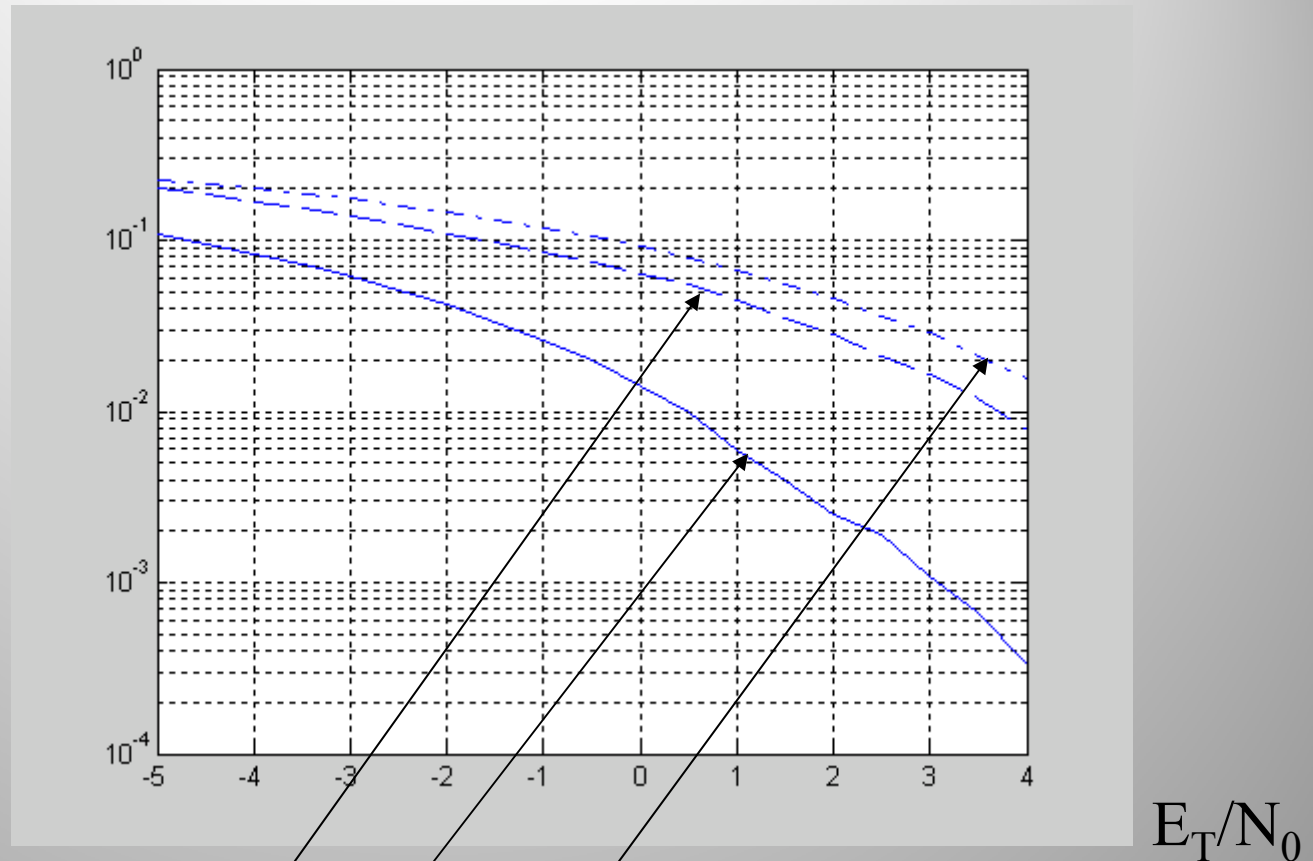
Since $\underline{\underline{B}}$ Is unitary, it has to be full rank, i.e. Size n_T by n_T (at least)

One symbol for n_T channel uses $\rightarrow$ Rate $1/n_T$

It is possible to increase this rate??

SPACE-TIME BLOCK CODES

BER



BER versus SNR for a MIMO system without CSIT. De abajo a arriba: MIMO(4,4) six streams and four PCUs ($r=3/4$); MIMO(3,3); MIMO(2,2) four streams and 2 PCUs ($r=1$).

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