

From point to point to Multiuser MIMO

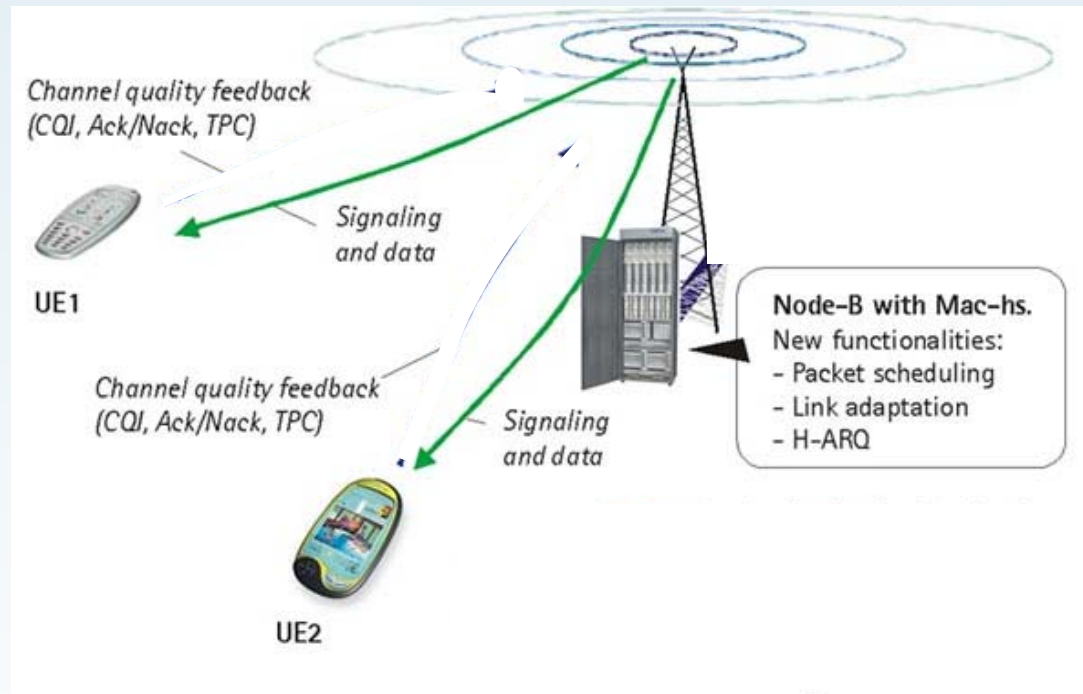
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- ▶ Introduction
- ▶ Lessons learned from Information Theory
 - ▼ MAC channel
 - ▼ BC channel
 - ▼ Duality
 - ▼ Summary
 - ◀ *Scaling laws of BC-MIMO (pre-log factor)*
 - ◀ *Information theory design guidelines*
- ▶ BC channel: signal processing design
 - ▼ Aiming at DPC: Linear and non-linear precoders
 - ▼ Practical schemes: Multiuser diversity
 - ◀ Opportunistic beam forming schemes
- * System issues

Multuser MIMO Capacity Regions

A very interesting scenario to exhibit the benefits of MIMO is the multuser scenario, where a single Base Station (BS) communicates with several geographically separated users. **Broadcast channel**



There is also the dual, the MAC (Multiple access) channel

Multiuser makes certain things difficult:

- Dealing with users of unequal channel conditions (fairness issues)
- Mixing antenna filtering and scheduling problems into a harder problem
- Multiple users cannot cooperate as well as multiple antennas on a single device
- Leads to multiple (rather than single) power constraints
- In BC, in the absence of CSIT, user multiplexing is generally not possible

But others much easier: (we have the user dimension)

- Provides multi-user diversity (less reliance on antenna diversity)
- Provides decorrelation of spatial signatures
- Allows for user- (in addition to stream -) multiplexing with low complexity receivers
- Low rank channels no longer a problem but an advantage

The BC is the most challenging

... because of downloads bottleneck

- Linear and non-linear precoding
- Channel State Information feedback
- Multiuser receivers
- User selection and Scheduling strategies
- Power control and other radio resource management

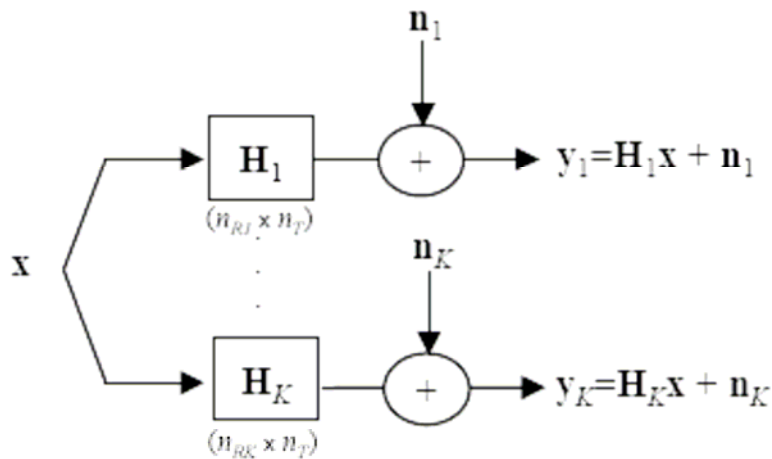
• Different sights

- From information theory point of view
- From signal processing point of view
- From network/protocol point of view

In single user SISO $C = \log(1 + SNR) \quad [bps / Hz]$

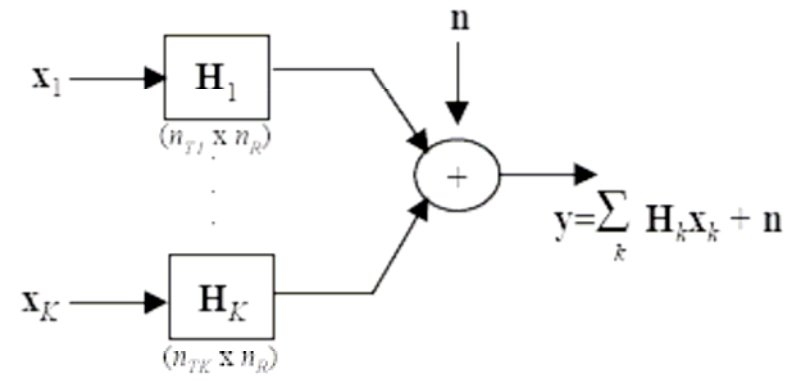
In MU-MIMO:

BS Shared power constraint



Broadcast Channel (BC)

UE individual power constraints



Multiple Access Channel (MAC)

$$C = \sum_k \log(1 + SNIR_k) \quad [bps / Hz]$$

?

$$y = Hs + w \Rightarrow C = \max_{P_x} I(s, y) = \max_{P_x} (H(y) - H(y/s)) =$$

$$= \log \frac{|R_y|}{|R_{y/x}|} = \log \frac{|R_y|}{|R_w|}$$

$$\det(A.B) = \det(A). \det(B)$$

$$\det(I_m + AB^T) = \det(I_n + B^T A)$$

$$\det(I + ab^T) = 1 + b^T a$$

$$\det(X + ab^T) = \det(X) (1 + b^T X^{-1} a)$$

$$\det(\exp(A)) = \exp(\text{tr}(A))$$

$$n = 1 \Rightarrow \det(A) = \text{tr}(A)$$

$$n = 2 \Rightarrow \det(A) = \frac{1}{2} (\text{tr}(A)^2 - \text{tr}(A^2))$$

$$d \det(A) = \det(A) \cdot \text{Tr}(A^{-1} dA)$$

$$\det(I + \varepsilon X) = 1 + \text{tr}(X) + O(\varepsilon^2)$$

In the cooperative MIMO case

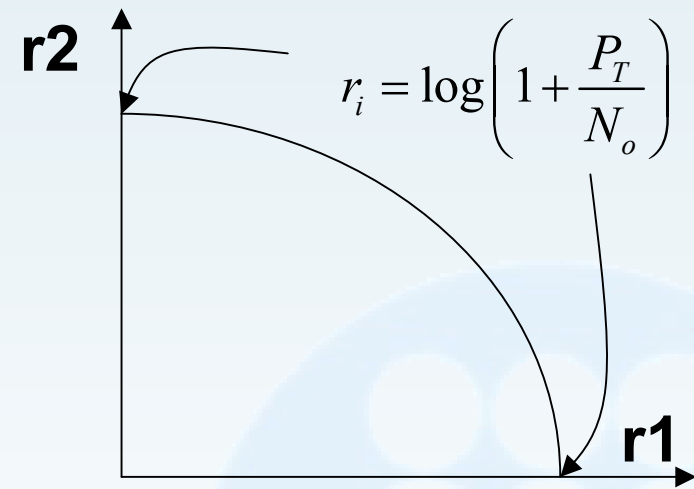
$y = Hs + w$ with $K = 2$ streams

$$\begin{cases} r_1 + r_2 < \log |I + \Lambda P| = \sum \log(1 + P_i \lambda_i) = \log(1 + P_1 \lambda_1 + P_1 \lambda_2 + P_1 P_2 \lambda_1 \lambda_2) \\ P_1 + P_2 = P_T \end{cases}$$

$$P_1 \lambda_1 = \exp(r_1) - 1$$

$$(\exp(r_1) - 1) / \lambda_1 + (\exp(r_2) - 1) / \lambda_2 = P_T$$

$$r_1 = \log(1 + P_T \lambda_1 - (\exp(r_2) - 1) \lambda_1 / \lambda_2)$$



$$\text{If } \lambda_1 = \lambda_2 \Rightarrow r_1 = \log(2 + P_T - \exp(r_2)) \Leftarrow \exp(r_1) + \exp(r_2) = P_T + 2$$

Sato's upper capacity bound

$$\lim_{SNR \rightarrow \infty} \frac{C_{MIMO}}{\log SNR} = \min(M, N)$$

There may be several “capacities” in multi-user tx.,
Unlike single-user tx.

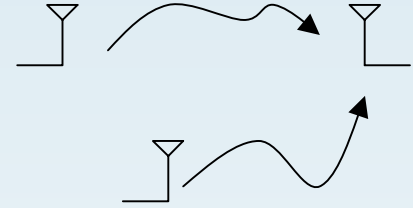
The **capacity region** is the tool in multi-user tx.

- It is the largest union (or convex hull) of all bps vectors
- Any point outside this region results in at least one of the receivers having a BER bounded away from 0 no matter what coding is used

For the single antenna case

K=2 users

$$y = \sum_i h_i s_i + w = \begin{pmatrix} h_1 & h_2 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + w$$



$$r_1 < \log \left(1 + \frac{P_1 |h_1|^2}{N_o} \right) = I(s_1; y | s_2)$$

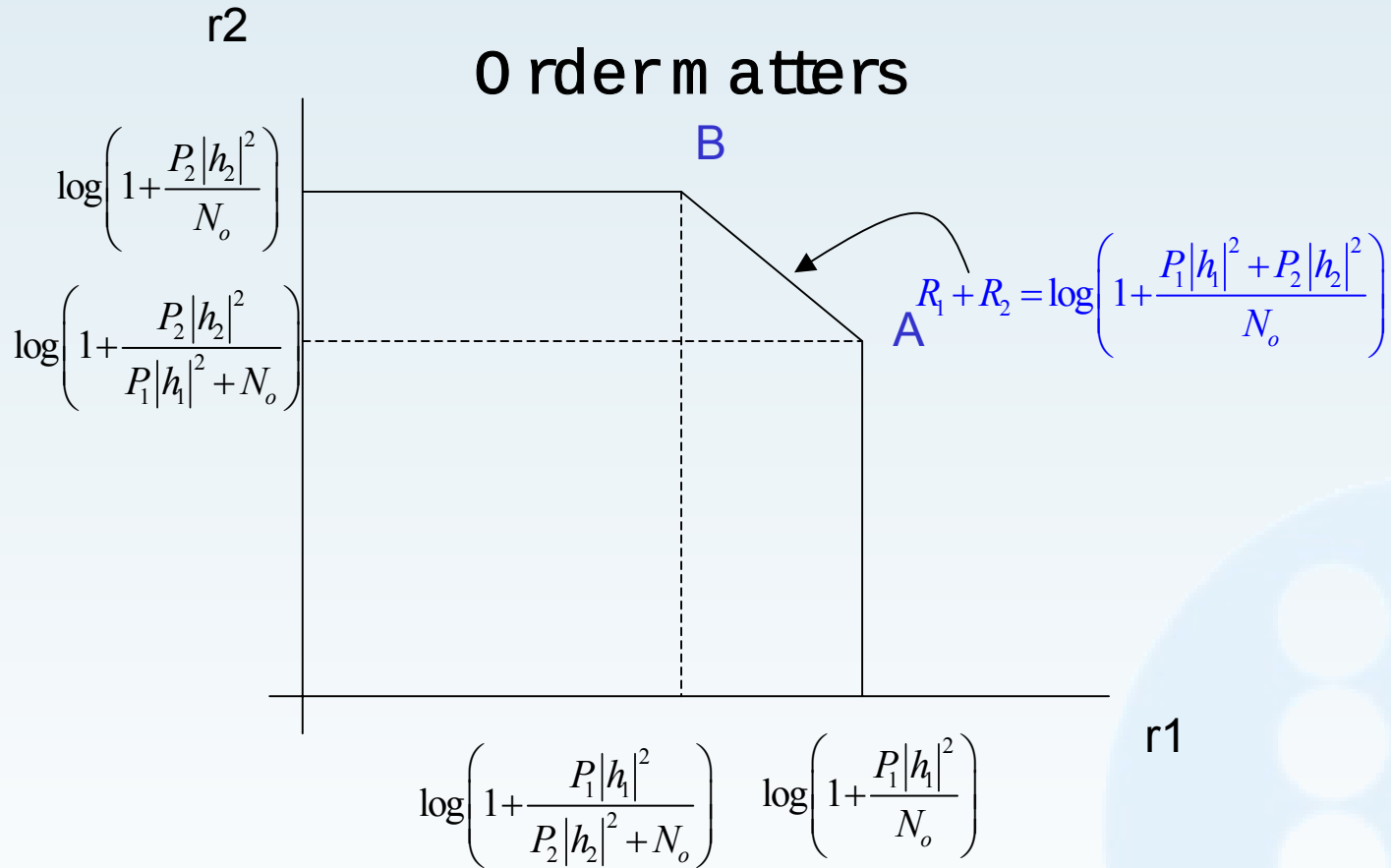
$$r_2 < \log \left(1 + \frac{P_2 |h_2|^2}{N_o} \right) = I(s_2; y / s_1)$$

$$r_1 + r_2 < \log \left(1 + \frac{P_1 |h_1|^2 + P_2 |h_2|^2}{N_o} \right) =$$

$$= \log \left(1 + \frac{P_2 |h_2|^2}{N_o} \right) + \log \left(1 + \frac{P_1 |h_1|^2}{P_2 |h_2|^2 + N_o} \right) = I(s_2; y / s_1) + I(s_1; y)$$

Chain rule, SIC decoder, K! comers

Capacity region

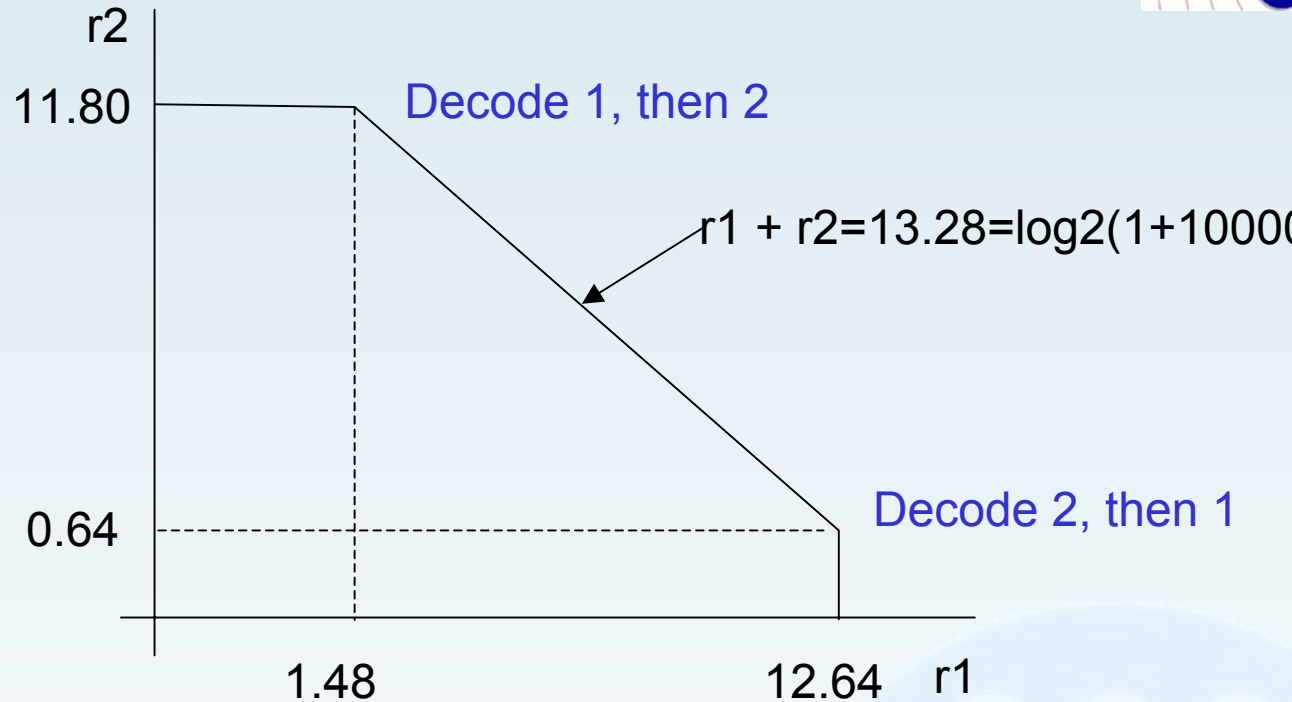


Exam p'le:

$$h_1=0.8$$

$$h_2=0.6$$

$$P_1=P_2=1$$



First alternative

$$r_2 = \log_2 \left(1 + \frac{.36 \cdot 1}{.0001 + .64} \right) = .64$$

$$r_1 = \log_2 \left(1 + \frac{.64 \cdot 1}{.0001 + .36} \right) = 1.48$$

Optimal alternative with SIC

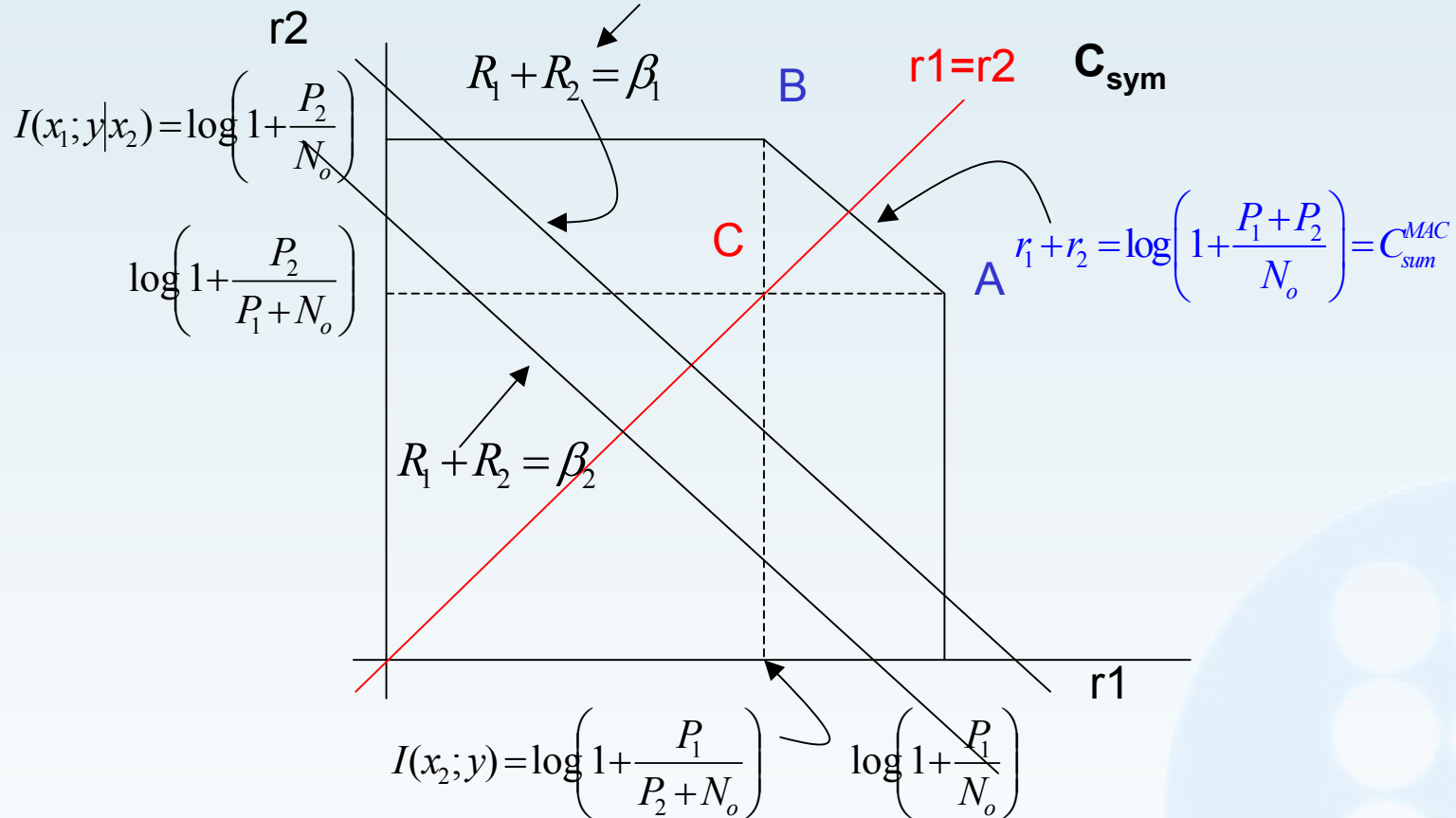
$$r_2 = \log_2 \left(1 + \frac{.36 \cdot 1}{.0001} \right) = 12.64$$

$$r_1 = \log_2 \left(1 + \frac{.36 \cdot 1}{.0001} \right) = 11.8$$

Capacity region

TDMA

$$C = \beta r_1 + (1 - \beta) r_2$$



FDMA:

$$r_1 < \beta \log\left(1 + \frac{P_1}{\beta N_o}\right) \quad r_2 < (1 - \beta) \log\left(1 + \frac{P_2}{(1 - \beta) N_o}\right)$$

The best strategy:

All users should access simultaneously

Note that at low SNR: power limited

K users

Capacity region C: set of all achievable rate vectors

$$C = \left\{ \sum_k \theta_k r_k; \theta_k \leq 1, \sum_k \theta_k = 1, r_k \in S, k \in \{1 \dots K\} \right\} = \text{convex hull} \{S\}$$

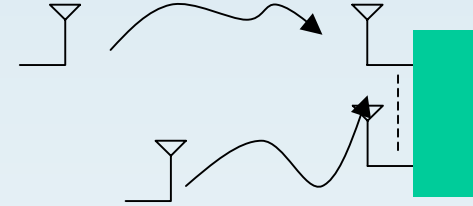
$$S = \bigcup_{P_i \leq P_T} \left\{ \sum_k r_k \leq \log \left(1 + \frac{\sum_i P_i |h_i|^2}{N_o} \right), k \in \{1 \dots K\} \right\}$$

Practical aspects: - chose decoding order
- chose subset of users S

For the multiple antenna case

K=2 users

$$\underline{y} = \begin{pmatrix} \underline{h}_1 & \underline{h}_2 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \underline{w} = Hs + w$$



$$r_1 < \log \left(1 + \frac{P_1 |\underline{h}_1|^2}{N_o} \right)$$

$$r_2 < \log \left(1 + \frac{P_2 |\underline{h}_2|^2}{N_o} \right)$$

$$SNIR_1 = SNR_1 = \frac{P_1 |\underline{h}_1|^2}{N_o}$$

$$SNIR_2 = \frac{P_2 |\underline{h}_2|^2}{N_o}$$

$$\underline{h}_2 = R_n^{-1/2} \underline{h}_2$$

$$r_1 + r_2 < \log \left| I + \frac{1}{N_o} H R_s H^H \right| = \log \left| I + \frac{1}{N_o} \sum_{i=1}^2 P_i \underline{h}_i \underline{h}_i^H \right|$$

$$\stackrel{R_s = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}}{=} \log \left(1 + \frac{P_1 |\underline{h}_1|^2 + P_2 |\underline{h}_2|^2}{N_o} + \frac{P_1 P_2}{N_o} |H H^H| \right) =$$

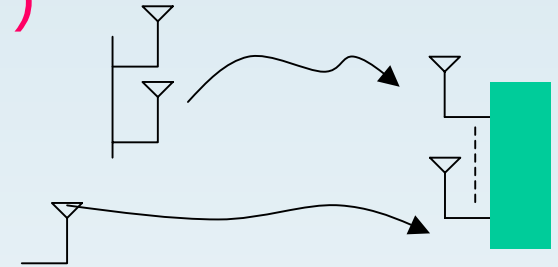
$$= \log \left(1 + \frac{P_2 |\underline{h}_2|^2}{N_o} \right) + \log \left(1 + P_2 \underline{h}_2^H \left(N_o I + P_1 \underline{h}_1 \underline{h}_1^H \right)^{-1} \underline{h}_2 \right) =$$

$$= \sum_{i=1}^2 \log (1 + SNIR_i)$$

Interesting for practical designs

For the MU-MIMO (K users, N, M)

$$\underline{y} = \begin{pmatrix} H_1 & H_2 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \underline{w} = Hs + w$$



K users

$$r_k \leq \log \det \left(\mathbf{I} + \frac{1}{N_o} \mathbf{H} \mathbf{R}_k \mathbf{H}^H \right) \quad k = 1 \dots K$$

$$C_{sum}^{MAC} = \sum_{k=1}^K r_k \leq \log \det \left(\mathbf{I} + \frac{1}{N_o} \sum_{k=1}^K \mathbf{H} \mathbf{R}_k \mathbf{H}^H \right)$$

$$\text{Tr}(\mathbf{R}_k) \leq P_k$$

The weaker are interfering

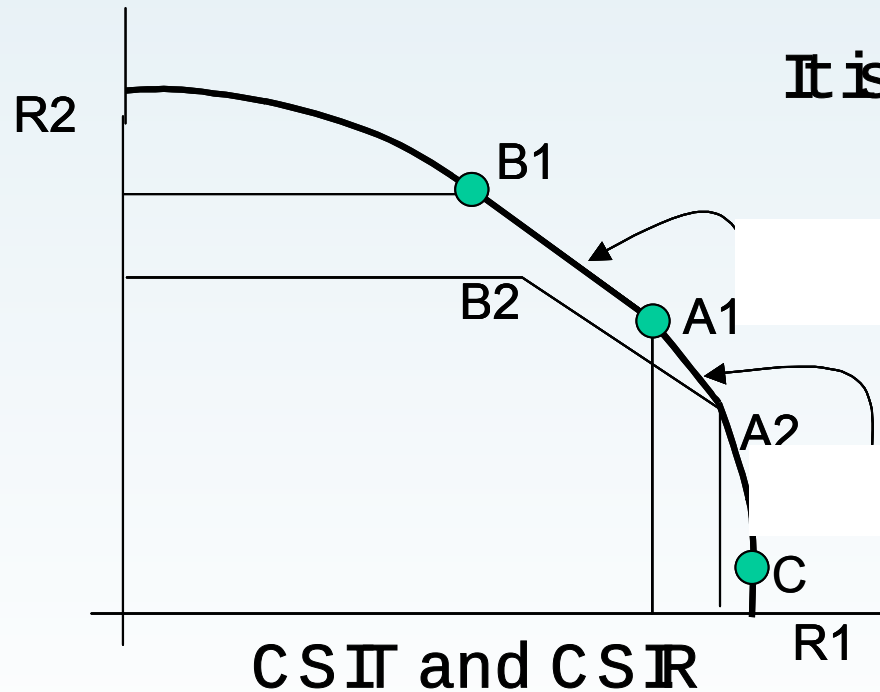
$$\sum_{i=1}^K r_i = \log \left| \mathbf{I} + \sum_{i=1}^K \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^H \right| = \log \left| \mathbf{I} + \mathbf{H}_1 \mathbf{R}_1 \mathbf{H}_1^H \right| + \dots + \log \frac{\left| \mathbf{I} + \sum_{i=1}^j \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^H \right|}{\left| \mathbf{I} + \sum_{i=1}^{j-1} \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^H \right|} + \dots + \log \frac{\left| \mathbf{I} + \sum_{i=1}^K \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^H \right|}{\left| \mathbf{I} + \sum_{i=1}^{K-1} \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^H \right|}$$

For the MU-MIMO

Capacity Region

$$C_{MAC} = \text{co} \left\{ \bigcup_{\{Tr(\mathbf{R}_i) \leq P_i \forall i\}} \left\{ (r_1 \dots r_K) : \sum_{i \in S} r_i \leq \log \left| I + \sum_{i \in S} \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^H \right| \quad \forall S \subseteq \{1 \dots K\} \right\} \right\}$$

It is concave on R_i

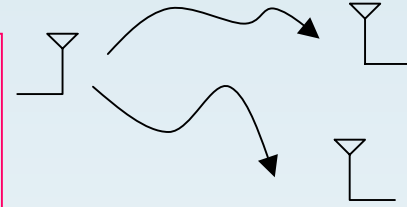


$$C_{MIMO-MAC}^{SUM} \approx \phi_0 \cdot \min \left[n_R, \sum_{k=1}^K n_{T_k} \right]$$

$$\frac{C_{MIMO-MAC}^{SUM}}{C_{TDMA}} \approx \min \left[\frac{n_R}{n_T}, K \right]$$

For the single antenna case

$$y_i = h_i s + w_i \Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \underbrace{(s_1 + s_2)}_x + w$$



K=2 users

Degraded channel

Symmetric case: $h_1=h_2=h$

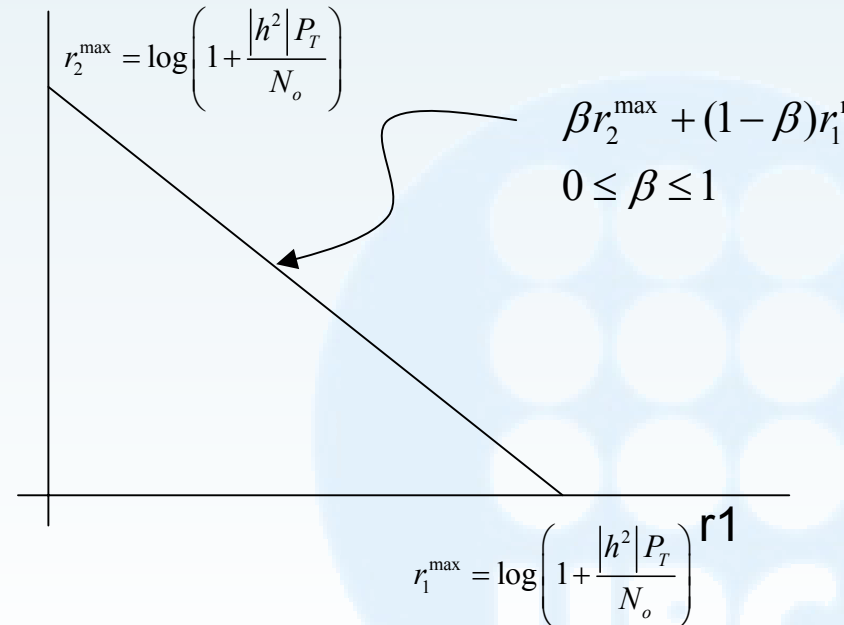
$$r_1 + r_2 < \log \left(1 + \frac{(P_1 + P_2)|h|^2}{N_o} \right)$$

$$P_2 + P_1 = P_T$$



$$r_2 = \log \left(1 + \frac{P_T}{N_o} |h|^2 \right) - r_1$$

r_2



For the single antenna case

More about r_1 and r_2 ?

Note that $y_1 = (s_1 + s_2)h + w_1$ $\sigma_1 < \sigma_2 \rightarrow w_2 = w_1 + w'$
 $y_2 = (s_1 + s_2)h + w_2$ $\rightarrow$ $y_2 = y_1 + w'$

y_2 is a degraded version of y_1

At each rx: the codeword intended for y_2 can also be decoded by y_1 and viceversa (SIC)

$$r_1 + r_2 < \log \left(1 + \frac{(P_1 + P_2)|h|^2}{N_o} \right) = \log \left(1 + \frac{P_2|h|^2}{P_1|h|^2 + N_o} \right) + \log \left(1 + \frac{P_1|h|^2}{N_o} \right)$$

For the single antenna case

If $|h_2| > |h_1|$

At rx 2: SIC, the performance of user 2 (the strongest one) is then

$$r_2 = \log \left(1 + \frac{P_2 |h_2|^2}{N_o} \right)$$

At rx 1 (weak): the weak user can only decode its own signal and user 2 acts as interference

$$r_1 = \log \left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_o} \right) = \log \left(1 + \frac{(P_1 + P_2) |h_1|^2}{N_o} \right) - \log \left(1 + \frac{P_2 |h_1|^2}{N_o} \right)$$

With respect to the MAC, users order is reversed: the strongest user has the better quality. Note that decoding order (1,2) produces smaller r_2 and User 1 would have less energy to use than with order 2,1

Fairness problem

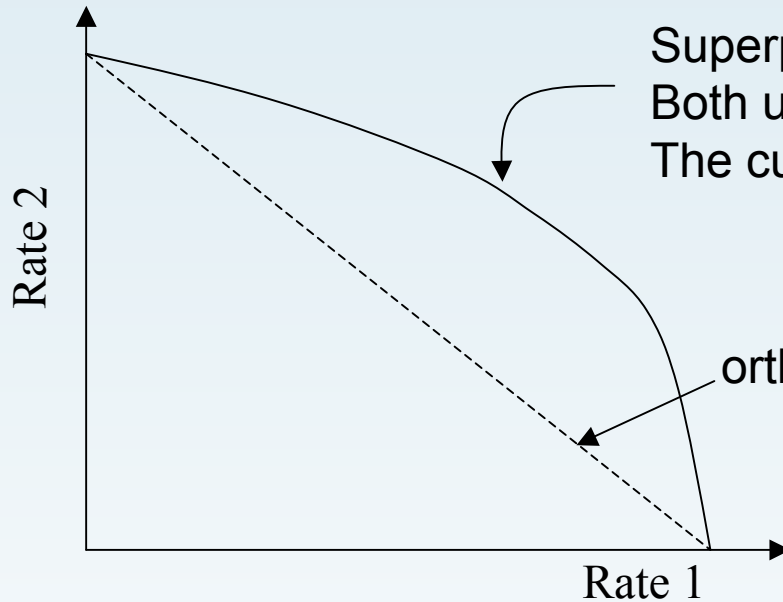
NOW, there is no closed expression for $r_1 + r_2$??

For the single antenna case

$$P_1 \in [1:100]$$

$$P_2 = 100 - [1:100]$$

By simulating both cases for



Superposition coding and asymmetric channels.
Both users transmit with total power constraint.
The curve is due to the global power constraint

$$r_1 = \log \left(1 + \frac{P_T (1 - \alpha) |h_1|^2}{P_T \alpha |h_1|^2 + N_o} \right) \quad \text{bits / s / Hz}$$

$$r_2 = \log \left(1 + \frac{\alpha P_T |h_2|^2}{N_o} \right) \quad \text{bits / s / Hz}$$

Best policy in BC-SISO if there is CSIT: one user at a time as we will see later on

However, SIC may present error propagation, therefore other multiuser detectors may be more practical or if CSIT is available Dirty Paper coding can be carried out, where s2 is coded in the already contaminated environment by s1

For the single antenna case

Other implementation: Dirty Paper Coding at tx

It is dual to SIC rx: interference cancellation at rx (CSIT)

$$tx_1 = s_1 \qquad tx_K = s_K - \sum_{i=1}^{K-1} cod(s_i)$$

$$tx_2 = s_2 - cod(s_1)$$

⋮



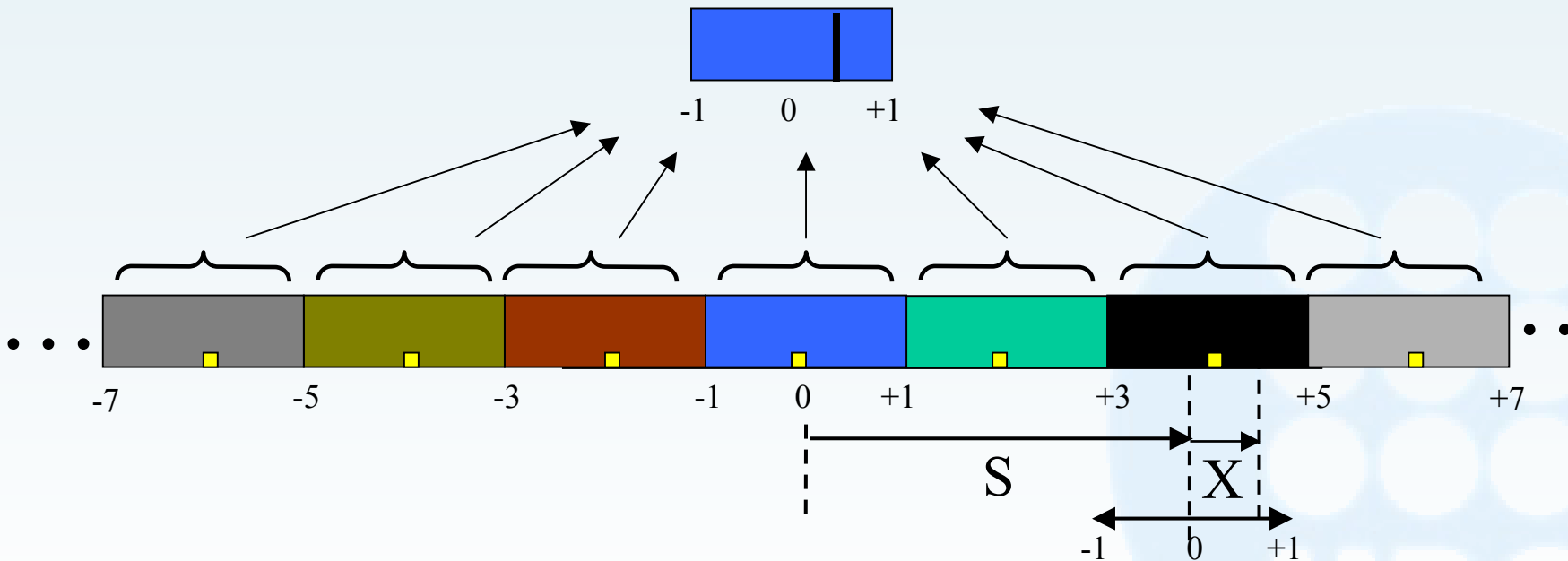
K=2

$$r_1 < \log \left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_o} \right) \quad \text{bits / s / Hz}$$

$$r_2 < \log \left(1 + \frac{P_2 |h_2|^2}{N_o} \right) \quad \text{bits / s / Hz}$$

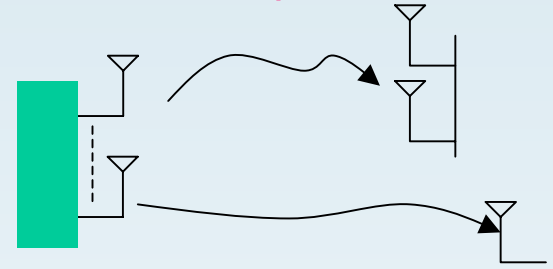
For the single antenna case. Example of DPC

- ▶ Received signal $Y = X + S$, $-1 \leq X \leq 1$
 - ▼ S known to transmitter, not receiver
- ▶ Modulo operation removes the interference effects
 - ▼ Set X so that $\lfloor Y \rfloor_{[-1,1]} = \text{desired message (e.g. 0.5)}$
 - ▼ Receiver demodulates modulo $[-1,1]$



For the multiple antenna case (K users, N, M)

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} H_1^H \\ H_2^H \end{pmatrix} \underbrace{(s_1 + s_2)}_x + w = H^H x + w$$



With CSIT and CSIR

$$r_i(\pi, R_k) = \log \frac{\left| \mathbf{H}_{\pi(i)} \left(\sum_{k=i}^K \mathbf{R}_{\pi(k)} \right) \mathbf{H}_{\pi(i)}^T + \mathbf{I} \right|}{\left| \mathbf{H}_{\pi(i)} \left(\sum_{k=i+1}^K \mathbf{R}_{\pi(k)} \right) \mathbf{H}_{\pi(i)}^T + \mathbf{I} \right|}$$

Non-convex



Difficult to obtain

$$\text{Tr} \left(\sum_i R_i \right) \leq P_T$$

The stronger are interfering

For the multiple antenna case

1.-

$$x = \sum_k W_k s_k$$

$$y_k = H_k W_k s_k + \sum_{j \neq k} H_j W_j s_j + n_k$$

$$R_k = W_k E \{ s_k s_k^H \} W_k^H$$

For W_k or rank 1 ($N=1$) $x = \sum_k w_k s_k$

$$r_i < \log \left(1 + \frac{P_i |h_i^H w_i|^2}{N_o + h_i^H \left(\sum_{j \neq i} P_j w_j w_j^H \right) h_i} \right) = \log (1 + SNIR_i^{DPC})$$

$$C_{sum}^{DP} = \sum_{i=1}^K \log (1 + SNIR_i^{DP})$$

Optimal order

Optimal user selection

Starting point for subopt. Strategies

- ▶ MAC capacity region known for many cases
 - ▼ Convex optimization problem
- ▶ BC capacity region typically only known for (parallel) degraded channels
 - ▼ Formulas often not convex
- ▶ Can we find a connection between the BC and MAC capacity regions?

Duality

For the multiple antenna case

2.- By duality

$$C_{sum}^{BC} = C_{sum}^{DP} = \sum_{i=1}^K R_i = \max_{\text{Tr}\left(\sum_k R_k^{MAC}\right)} \log \left| \mathbf{I} + \sum_{i=1}^K \mathbf{H}_i^H \mathbf{R}_i^{MAC} \mathbf{H}_i \right|$$

For the multiple antenna case

3.- For the Gaussian case

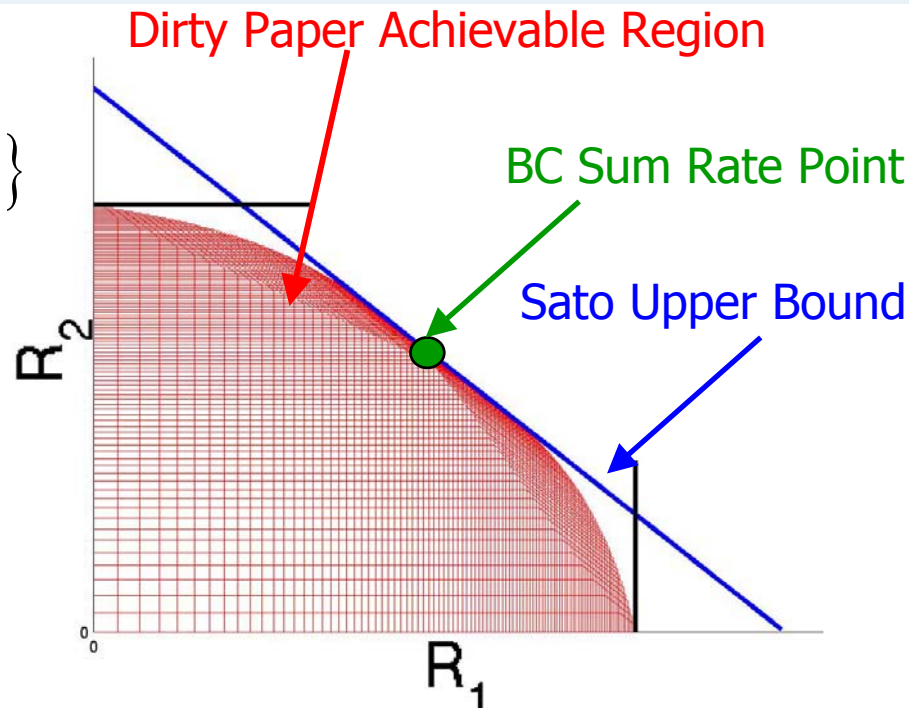
$$C_{BC}(P, \mathbf{H}) = Co \left(\bigcup_{\pi, \mathbf{R}} r(\pi, \mathbf{R}) \right)$$

In addition, by duality

$$C_{BC}(\bar{P}, \mathbf{H}) = \bigcup_{\{P_i\}_i^K: \sum_{i=1}^K P_i \leq \bar{P}} C_{MAC} \{ \mathbf{R}_1, \dots, \mathbf{R}_K; \mathbf{H}^H \}$$

Flip the channel and
reverse the order

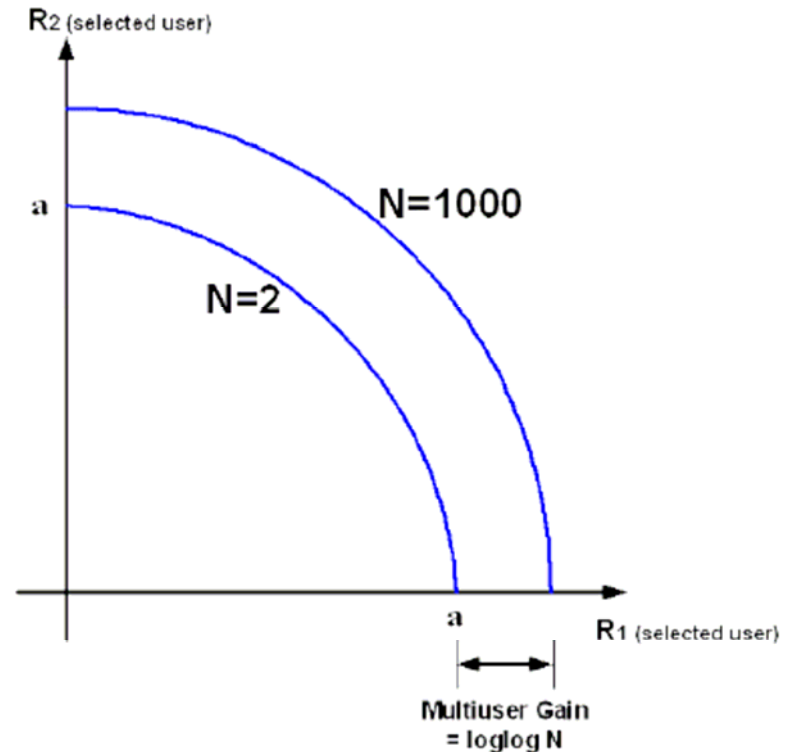
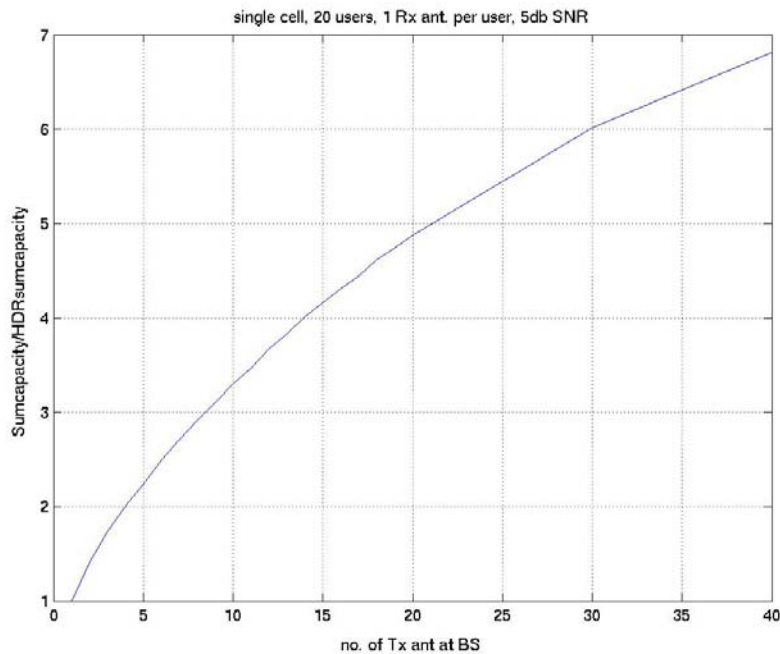
The union of convex reg.



For the multiple antenna case

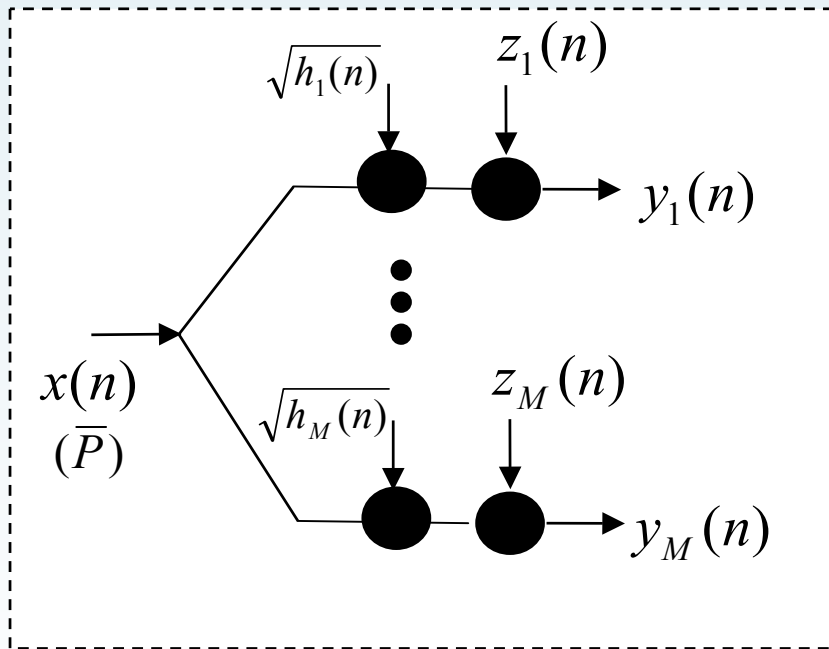
4.- Effect of K

DPC can be applied over $K > n_t$

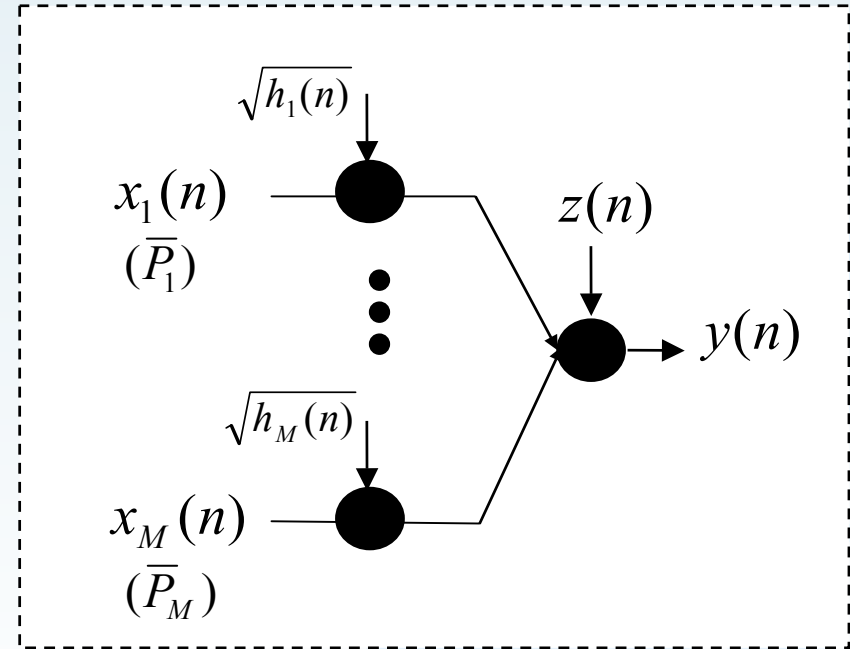


Dual Broadcast and MAC Channels

Gaussian BC and MAC with *same* channel gains and *same* noise power at each receiver



Broadcast Channel (BC)

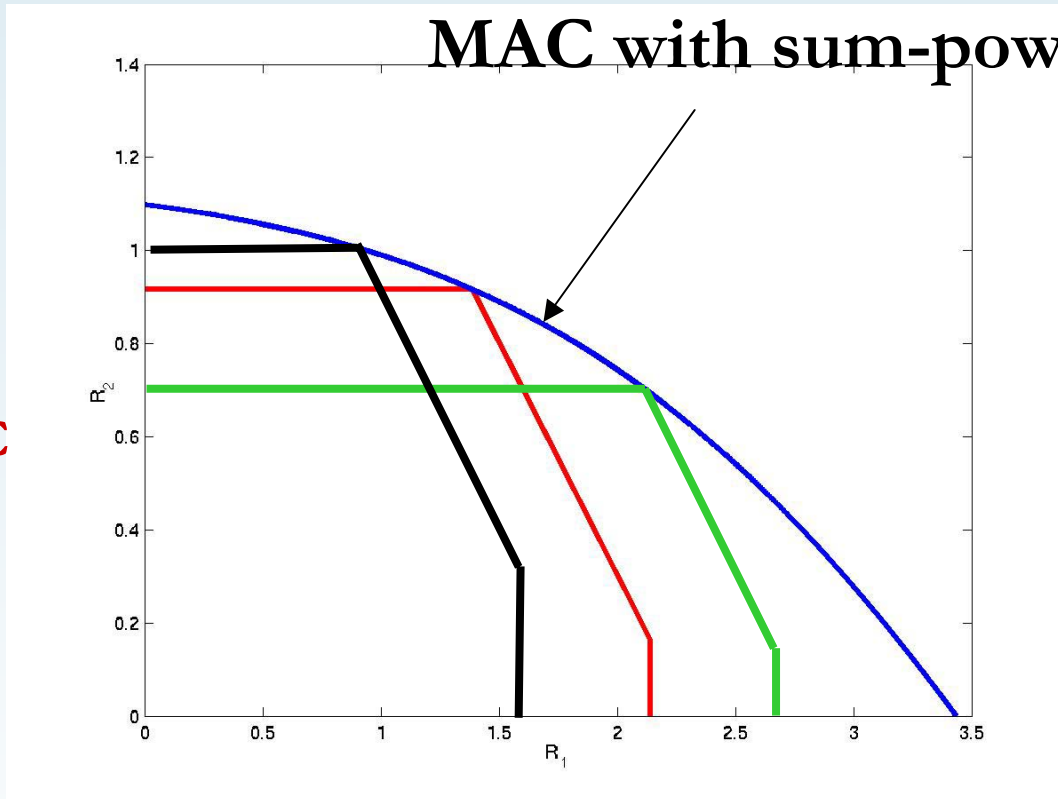


Multiple-Access Channel (MAC)

$$C_{MAC}(P_1, P_2; h_1, h_2) \subseteq C_{BC}(P_1 + P_2; h_1, h_2)$$

$$h_1 > h_2$$

Blue = BC
Red = MAC



$$P_1=0.5, P_2=1.5$$

$$P_1=1, P_2=1$$

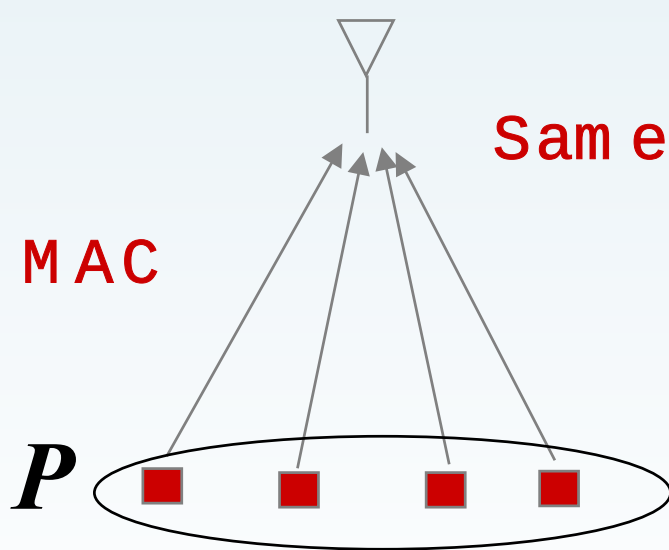
$$P_1=1.5, P_2=0.5$$

$$C_{BC}(P; h_1, h_2) = \bigcup_{0 \leq P_1 \leq P} C_{MAC}(P_1, P - P_1; h_1, h_2)$$

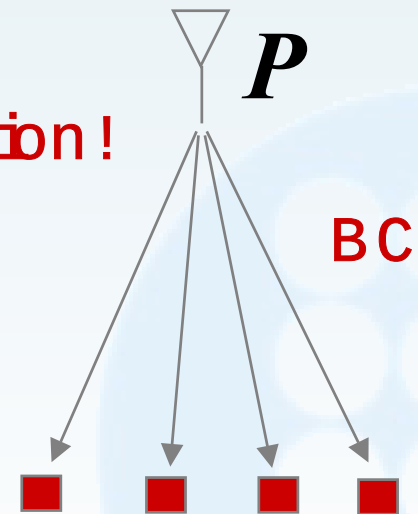
Sum Power MAC

$$C_{BC}(P; h_1, h_2) = \bigcup_{0 \leq P_1 \leq P} C_{MAC}(P_1, P - P_1; h_1, h_2) \equiv C_{MAC}^{Sum}(P; h_1, h_2)$$

- MAC with sum power constraint
 - Power pooled between MAC transmitters
 - No transmitter coordination



Same capacity region!



For the single antenna case

An Example

$$C_{BC}(P; h_1, h_2) = \bigcup_{0 \leq P_1 \leq P} C_{MAC}(P_1, P - P_1; h_1, h_2)$$

What is the relationship between the optimal strategies?

Equating rates and solve for powers:

$$r_1^{MAC} = \log \left(1 + \frac{|h_1|^2 P_1^{MAC}}{N_o + |h_2|^2 P_2^{MAC}} \right) = \log \left(1 + \frac{|h_1|^2 P_1^{BC}}{N_o} \right) = r_1^{BC}$$

$$r_2^{MAC} = \log \left(1 + \frac{|h_2|^2 P_2^{MAC}}{N_o} \right) = \log \left(1 + \frac{|h_2|^2 P_2^{BC}}{N_o + |h_2|^2 P_1^{BC}} \right) = r_2^{BC}$$

Opposite decoding order



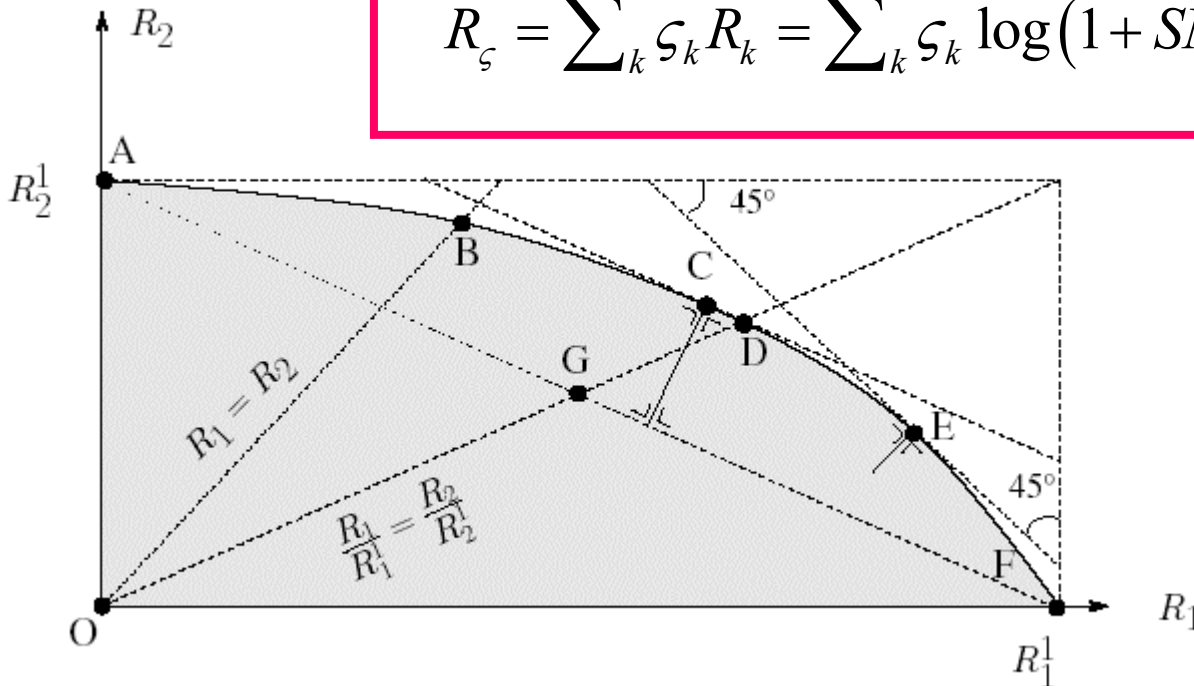
- ▶ Shannon capacity gives fundamental data rate in its for wireless channels
- ▶ Broadcast channels with ISI can use OFDM with near-optimality
- ▶ Duality and dirty paper coding are used to obtain the capacity of a broadcast MIMO channel

Interestingly, both capacity regions are exactly the same (duality property) as soon as the power constraint is set on the total transmitted power.

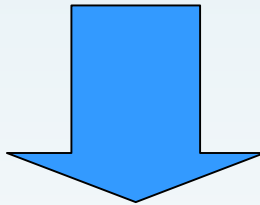
The boundary of the global capacity region can be traced out by means of a set of relative priority coefficients $\sum_k \varsigma_k = 1$

Each boundary point of the capacity region maximizes the linear combination of the user rates

$$R_\varsigma = \sum_k \varsigma_k R_k = \sum_k \varsigma_k \log(1 + \text{SNIR}_k)$$



The fundamental role played by the multiple antennas at either the BS or the users in expanding the channel capacity is best apprehended by examining how the sum rate scales with the number of users K



Capacity scaling laws in MIMO-BC or pre-log factor

Capacity scaling laws in MIMO-BC or pre-log factor

M tx, N rx, K users

Full CSIT and CSIR (in practice)

$$\lim_{SNR \rightarrow \infty} \frac{C_{DP}}{\log SNR} = \min(M, \max(N, K))$$

$$\lim_{K \rightarrow \infty} \frac{C_{DP}}{\log \log KN} = M$$

No CSIT and full CSIR
(TDMA)

$$\lim_{SNR \rightarrow \infty} \frac{C_{DP}}{\log SNR} = \min(M, N)$$

$$= \lim_{SNR \rightarrow \infty} \frac{C_{P2P-MIMO}}{\log SNR}$$

$$\lim_{K \rightarrow \infty} \frac{C_{DP}}{\log \log KN} = 0$$

the BS selects and tx only to the user with max rate

$$\lim_{K \rightarrow \infty} \frac{C_{TS}}{\log \log K} = \min(M, N)$$

Information theoretic design guidelines

- Capacity scaling laws advocate for SDMA. How many and which users should be served at any instant of time, and how much power, is the problem addressed by the scheduler and resource allocation strategy
- Unlike the P2P MIMO, spatial mux is possible with $N=1$
- The mux gain comes at the condition of cbsc to perfect CSIT, this is different from P2P MIMO, where the asymptotic capacity does not depend on the CSIT. therefore, the importance of the feedback channel
- Take advantage of Multiuser Diversity (big K)
- Precoder design together with scheduling: M users out of K

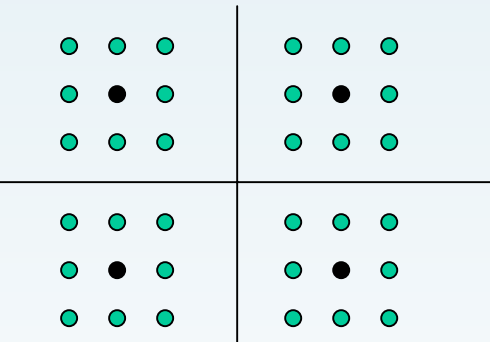
$$C = \sum_k \log(1 + SNIR_k) \quad [bps / Hz]$$

- ▶ Introduction
- ▶ Lessons learned from Information Theory
 - ▼ MAC channel
 - ▼ BC channel
 - ▼ Duality
 - ▼ Summary
 - ◀ *Scaling laws of BC-MIMO (pre-log factor)*
 - ◀ *Information Theory design guidelines*
- ▶ BC channel: signal processing design
 - ▼ Aiming at DPC: Linear and non-linear precoders
 - ▼ Practical schemes: Multiuser diversity
 - ◀ Opportunistic beam forming schemes
- ▶ System issues

For the single antenna case

In practice:

- 1.- For the general case: superposition coding achieves capacity
Superposition coding is a multiresolution technique



32-QAM con QPSK

$$\text{QPSK} \rightarrow R_1 = \log \left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_o} \right)$$

$$\text{32-QAM} \rightarrow R_2 = \log \left(1 + \frac{P_2 |h_1|^2}{N_o} \right)$$

For the single antenna case

In practice:

2.- The weaker user

$$r_1 = \log \left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_o} \right) \quad \text{bits / s / Hz}$$

The strongest user performs SIC at rx2

$$r_2 = \log \left(1 + \frac{P_2 |h_2|^2}{N_o} \right) \quad \text{bits / s / Hz}$$

For the single antenna case

On the other hand, orthogonal schemes achieve, for each power split $P = P_1 + P_2$ and degree-of-freedom split a

$$R_1 = \beta \log \left(1 + \frac{P_1 |h_1|^2}{\beta N_o} \right)$$

$$R_2 = (1 - \beta) \log \left(1 + \frac{P_2 |h_2|^2}{(1 - \beta) N_o} \right)$$

One can show that superposition coding is strictly better than the orthogonal schemes. In these ones, a significant fraction of the degrees of freedom to the weak user are needed to achieve near single-user performance, degrading the strong user