



A in ing atDPC:Linear and non-linearprecoders





Master Merit



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Linear and non-linear precoders



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Opportunistic beamforming









•A smart beam generation policy can improve the performance of the opportunistic schemes in outdoor scenarios with limited number of users.

•A power allocation over the transmitted beams also enhances the performance of MOB.

•To further boost the efficiency of MOB, a progressively full CSIT from the scheduled users can be used to obtain a triangular interference cancellation.

The ability of accurately predicting the channel SNR dominates the performance

of opportunistic beamforming Other alternatives for CSIT

Other alternatives for CSIT and precoding ?

IT IS AN UNSOLVED PROBLEM

In SU- MIMO: feedback of B

BUT in MU-MIMO: Bi (i=1...N) precoders that depend on Hj



Partial CSIT



IN PRACTICE IT IS A TWO STAGE PROBLEM

- 1. User selection: Decision making process
 - Signal Processing for opportunistic identification
 - System issues for opportunistic exploitation
- 2. Precoder design
- DIMENSION REDUCTION & PROJECTION TECHNIQUES

Projecting the matrix channel onto one or more basis vectors known to the tx and rx

Ex.: For densily populated areas

$$\varphi_k = \max_{i=1..nt} \frac{\left|h_k^H b_i\right|^2}{\sigma^2 + \sum_{i \neq j} \left|h_k^H b_j\right|^2}$$

- TEMPORAL STATISTICAL FEEDBACK: for low mobility
- SPATIAL STATISTICAL FEEDBACK: for outdoor



Partial CSIT



SPATIAL STATISTICAL FEEDBACK: for outdoor

Channel statistics (macroscopic information of the channel): $h_k \square CN(\overline{h}_k, R_k)$ Instantaneous information: Example

QUANTIZATION-BASED FEEDBACK

It is the first idea that comes into mind when thinking about source compression

Vector quantization entails designing a codebook that encapsulates the essential degrees of freedom of the channel and is tailored to the channel model and receiver design. A pure VQ approach would attempt to obtain a good approximation of a given channel realization; the goal of limited feedback communication, though, is to maximize capacity or minimize bit error rate with a few bits of feedback information.

The codebook of each user should be different from others. Otherwise, there is a chance that two or more users quantize their channel vectors to the same code vector, which will cause a rank loss in the quantized channel matrix composed by those code vectors. To avoid this situation, we let every user rotate a general codebook by a random unitary matrix that is also known at the base station so that the CSIT matrix is full rank with probability one.





User Selection



•From a multiuser information theoretic perspective: all K users should be served •M2 upperbounds the optimal number of users with non-zero allocated power •With linear precoders the #served users is limited by M (DoF at BS)

EXAMPLE 1: OPTIMAL USER SELECTION WITH BLOCK DIAGONALIZATION

Set of all users $U = \{1, 2, \dots, U\}$ $A = \{A_1, A_2, \dots\}$ $r_{BD/A_k} = \max_{\substack{\sum j \in A_k \\ j \in A_k}} \log \left| I + \frac{H_j B_j Q_j B_j^H H_j^H}{\sigma^2} \right| \quad card(A) = \sum_{i=1}^{3} C_U^i$ $C_{BD} = \max_{A_k \in A} r_{BD/A_k}$



Max. # users tb sup.

O(U^K)



User Selection



EXAMPLE 2: GREEDY USER SELECTION

- Capacity-based greedy user selection ? UxK user sets At each step, re-processing of the linear beamforming (if joint scheduling & beamf.)
- Semi-orthogonal selection

$$\left|h_{k}^{H}h_{j}\right|\leq\varepsilon$$





Problem Statement



Consider linear precoders and decoders

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{B}_k \mathbf{s}_k + \sum_{j \in S, j \neq k} \mathbf{H}_k \mathbf{B}_k \mathbf{s}_j + \mathbf{w}_k$$

First, we address the problem of transmit beamforming $B_k = b_k$

The general SNIR is

$$\gamma_k = \frac{\mathbf{b}_k \mathbf{R}_k \mathbf{b}_k^{\mathrm{H}}}{\sum_{i \neq k} \mathbf{b}_i \mathbf{R}_k \mathbf{b}_i^{\mathrm{H}} + \sigma_k^2}$$

The optimal beamforming strategy in terms of rate is

$$R_{BF} = \max_{b_k, P_k} \sum_{i=1}^{Ntot} \log(1 + SNIR^{BF})$$

s.t. $\sum_{i=1}^{Ntot} |\mathbf{b}_k|^2 P_k \le P$

But it is difficult to carry out in practice: SNIRk depends on the other users' bj



Problem Statement



The transmit precoding optimization problem can be approached under different assumptions, such as power constraints (total or individual), and with different performance criteria (e.g. max. SINR, sum rate, BER,...) The difficulty of designing capacity-optimal downlink precoding, mainly due to The coupling between power and beamforming and the user ordering, has lead To several different approaches ranging from transmit power minimization with SNIR constraints to worst case SINR max. Under power constraint. Duality and Iterative algorithms are often used in order to provide solutions.

The optimal beamforming strategy in terms of rate is

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Duality: target SNIR



Consider B for BC and B^H for MAC Consider $\begin{bmatrix} \mathbf{B}\mathbf{B}^H \end{bmatrix}_{jj} = 1 \longrightarrow \sum_k P_k^{bc} \leq P$

In the BC we have

$$SINR_{bc,i} = \frac{P_i^{bc} \phi_{ii}}{1 + \sum_{j \neq i} P_j^{bc} \phi_{ij}} \qquad \phi_{ij} = |\mathbf{HB}|$$

In the MAC we have

$$SINR_{mac,i} = \frac{P_i^{mac}\phi_{ii}}{1 + \sum_{j \neq i} P_j^{mac}\phi_{ij}}$$

The system of equations $SINR_i \ge \gamma_i$

$$\mathbf{a} = \begin{bmatrix} a_1 \cdots a_1 \end{bmatrix}^T \qquad a_i = \frac{\gamma_i}{(1 + \gamma_i)\phi_{ii}} \\ \begin{bmatrix} \mathbf{I} - diag(\mathbf{a})\mathbf{\Phi} \end{bmatrix} \mathbf{p}^{bc} \ge \mathbf{a} \qquad \begin{bmatrix} \mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}^T \end{bmatrix} \mathbf{p}^{MAC} \ge \mathbf{a} \end{bmatrix}$$

 $|^2_{ii}$



Duality: target SNIR



$$\left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}\right]\mathbf{p}^{bc} \ge \mathbf{a} \qquad \left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}^{T}\right]\mathbf{p}^{MAC} \ge \mathbf{a}$$

The SINR vector γ is feasible for both BC and MAC with linear processing matrix B if and only if the non-negative matrix diag(a) Φ has Perron-Frobenious eigenvalue $\rho(diag(a)\Phi)<1$. In this case, the solutions are

$$\mathbf{p}_{opt}^{bc} = \left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}\right]^{-1}\mathbf{a} \qquad \mathbf{p}_{opt}^{mac} = \left[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}^{T}\right]^{-1}\mathbf{a}$$
Moreover $\sum_{i} p_{opt,i}^{bc} = \sum_{i} p_{opt,i}^{mac}$





Solutions for Assigned Target SINRs



The classical beamforming problem is

 $\min_{u_k, p_k} \sum_{k=1}^{K} p_k$ $s.t. \gamma_k \ge \gamma_{thres} \qquad k = 1...K$

Direct method for solution based on semidefinite programming Alternative based on up-down duality

In the uplink, the MMSE filter maximizes the SINR

$$\mathbf{B} = \left[\mathbf{I} + \mathbf{H}^{H} diag(\mathbf{p})\mathbf{H}\right]^{-1}\mathbf{H}^{H}\mathbf{A}$$

Then, the problem involves just minimizing over the power p

$$\min_{p} \sum_{i} p_{i}$$

s.t. $p_{i}h_{i}R_{ni}^{-1}h_{i}^{H} \ge \gamma_{i}$ with $R_{ni} = I + \sum_{j \ne i} p_{j}h_{j}^{H}h_{j}$



Solutions for Assigned Target SINRs



The problem

$$\min_{p} \sum_{i} p_{i}$$
s.t. $p_{i}h_{i}R_{ni}^{-1}h_{i}^{H} \ge \gamma_{i}$ with $R_{ni} = I + \sum_{j \ne i} p_{j}h_{j}^{H}h_{j}$

Belongs to the class of standard power control problems [Yates-JSAC95] Therefore, if the problem is feasible, the iterative power control algorithm is given by

$$p_{i}^{(l)} = \frac{\gamma_{i}}{h_{i} \left[I + \sum_{j \neq i} p_{j}^{(l-1)} h_{i}^{H} h_{i} \right]^{-1} h_{i}^{H}}$$

If the problem is feasible, MMSE B can be used in the BC and the powers can be obtained by solving

$$\big[\mathbf{I} - diag(\mathbf{a})\mathbf{\Phi}\big]\mathbf{p}^{bc} \geq \mathbf{a}$$



Solutions for Assigned Target SINRs



The problem of

 $\max_{B,q} \min_{i} SINR_{i}$ s.t. $\sum_{i} q_{i} \le P$

Is equivalent to

$$\min_{u_k,p_k}\sum_{k=1}^K p_k$$

 $s.t.\gamma_k \ge \gamma_{thres}$ k = 1...K

When the SINR target vector has all component equal to γ . In practice, in order to determine γ opt we shall solve for increasing values of γ , until the value of the resulting power sum crosses de level P





Maximum sumrate



Coming back to

$$\max_{\gamma_k} \sum_{i=1}^{Ntot} \log(1+\gamma_i)$$

s.t. $\gamma \in \Psi(P,H)$

We note that the constraint set of all SINR vectors such that the sum power is constrained is not convex. Applying duality, the problem keeps non-convex. Heuristic approaches for throughput maximization with linear beamforming have been proposed.





Maximum sumrate



Coming back to

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A standard suboptimal approach providing a promising tradeoff between complexity and performance is channel inversion or ZF-beamformer.

$$\min_{B} Tr(BB^{H})$$

s.t. $HB = I$ $B = H^{H}(HH^{H})^{-1}$

The achievable sum rate is given by

R

s.t.

$$r_{ZF} = \max_{\substack{\sum q_k q_k \le P \\ k \in S}} \sum_{k \in S} \log(1 + q_k)$$

$$\eta_k = \frac{1}{\|b_k\|^2} = \frac{1}{\left[\left(HH^H\right)^{-1}\right]_{kk}}$$
 Very appealing

If the objective is to maximize the sum_rate, the optimum power allocation is

$$q_{k} = \eta_{k} \left[\mu - \frac{1}{\eta_{k}} \right] \qquad \forall k \in S$$

$$\sum_{k \in S} \left[\mu - \frac{1}{\eta_{k}} \right]^{+} = P$$

$$r_{ZF} = \sum_{k \in S} \left[\log(\mu \eta_{k}) \right]^{+} \qquad \text{Ext}$$

haustive search is needed

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•Note that the sumrate of channel inversion without user selection does not Increase linearly with M, unlike capacity. User selection gives an important degree of freedom by selecting group of users with mutually orthogonal spatial signatures. Then, for large K, ZFBF with user selection is shown to achieve both mux and mud gain.

•User selection is still an open problem. It is related to the following geometrical Problem: Given a set of K vectors find an optimal "self-basis" such that the

Gram matrix HH^H has maximum determinant.













Technique	Gain	Mean	Standard Deviation	Asymptotic IF
Cooperative	λ_k^2/K	Q/K	$\sqrt{Q/K}$	$1/(1+\xi)$
Dirty Paper	d_k^2/K	(2Q-K+1)/2K	$\sqrt{Q+\tfrac{1}{12}(K-5)(K-1)}/K$	$(2-\xi)^2/\left[(2-\xi)^2+\xi^2/3\right]$
Zero Forcing	α_k^2/K	(Q-K+1)/K	$\sqrt{Q-K+1}/K$	1

Observe that ZF is not optimal, DPC tells us that it is beneficial to allow some interference at the receiver to increase the received power of the desired signal

In general

$$B = \beta H^{H} (HH^{H})^{-1}$$

with $\beta = \sqrt{\frac{P}{Tr((HH^{H})^{-1})}}$







Point to Multipoint





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MMSE precoder



For rank deficient channels, the performance of the ZF can be improved by regularization of the pseudo-inverse

$$B = H^{H} \left(H H^{H} + \lambda I \right)^{-1}$$

More specifically

y = HBs + w $\min_{B,\beta} E \|s - \beta^{-1}y\|^{2}$ S.t. $E \|Ps\|^{2} = P$ But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonal<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalBut the MMSE does not provide parallel and orthogonalBut the MMSE does not provide parallel and orthogonalChannels and thus power allocation are not straightforward<math display="block">But the MMSE does not provide parallel and orthogonalBut the MMSE does not provide parallel and orthogonal<math display="block">But the MMSE does not provide parallel and orthogonalBut the MMSE does not parallel and orthogonal<math display="block">But the MMSE does not parallel and orthogonalBut the MMSE does not parallel and orthogo

Note that at low SNR it is equivalent to the Matchef filter transmitter



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MMSE decoder





$$I(X;Y) = I(Y;X) = H(X) - H(X/Y) = \frac{1}{2}\log\frac{|\mathbf{R}_x|}{|\mathbf{R}_{x/y}|} = \frac{1}{2}\log\frac{|\mathbf{R}_x|}{|\mathbf{R}_e|}$$

e is not a white vector, therefore

$$\frac{1}{2}\log\frac{|\mathbf{R}_{x}|}{|\mathbf{R}_{e}|} \geq \frac{1}{2}\log\frac{|\mathbf{R}_{x1}|}{|\mathbf{R}_{e1}|} + \frac{1}{2}\log\frac{|\mathbf{R}_{x2}|}{|\mathbf{R}_{e2}|}$$
$$|\mathbf{R}_{e}| \leq |\mathbf{R}_{e_{1}}||\mathbf{R}_{e_{2}}|$$

Independent decoding of each stream is capacity lossy !!!

Therefore, by duality MMSE precoder is capacity lossy, because it assumes independent decoding of each stream

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The goal of the DFE is to use a decision-feedback structure to enable the independent decoding of x1 and x2. This is accomplished by a diagonalization of the MMSE error e, while preserving the "information" in xm.

The diagonalization of the MMSE error can be done via a Block Cholesky factorization as follows

$$\mathbf{R}_{e} = \mathbf{G}^{-1} \boldsymbol{\Delta}^{-1} \mathbf{G}^{-\mathrm{T}} \qquad \mathbf{G} = \begin{bmatrix} \mathbf{I} & \mathbf{G}_{22} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \qquad \boldsymbol{\Delta} = \begin{bmatrix} \boldsymbol{\Delta}_{11} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Delta}_{22} \end{bmatrix}$$

Then

$$\mathbf{x} = \mathbf{A}\mathbf{y} + \mathbf{e} =$$
$$= \mathbf{G}^{-1} \mathbf{\Delta}^{-1} \mathbf{G}^{-T} \mathbf{H}^{T} \mathbf{y} + \mathbf{e}$$

In order to decouple the error

$$G\mathbf{x} = \Delta^{-1} \mathbf{G}^{-T} \mathbf{w} + \mathbf{G} \mathbf{e} = \Delta^{-1} \mathbf{G}^{-T} \mathbf{w} + \mathbf{G} \mathbf{e} \qquad (*)$$
$$\mathbf{e}' = \mathbf{G} \mathbf{e} = \begin{pmatrix} \mathbf{I} & \mathbf{G}_{22} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{pmatrix} \rightarrow \mathbf{R}_{\mathbf{e}'} = \Delta^{-1}$$
$$|\mathbf{R}_{\mathbf{e}'}| = |\Delta_{11}^{-1}| |\Delta_{22}^{-1}|$$





Thus e' is uncorrelated. From equation (*) we get

 $\mathbf{x} = \Delta^{-1}\mathbf{G}^{-T}\mathbf{w} + (\mathbf{I} - \mathbf{G})\mathbf{x} + \mathbf{e'} = \mathbf{x'} + \mathbf{e'}$

Which gives the DFE structure of the new receiver shown in the figure, where the feedback filtering part can be implemented a successive interference cancellation due to the triangular structure of G. Note that in case Re were factorized following the SVD decomposition, then the successive interference cancellation interpretation is lost.













The achievable rates are

$$R_{1} = I(X_{1}^{'}; X_{1}) = \frac{1}{2} \log \frac{|\mathbf{R}_{1}|}{|\mathbf{R}_{e'1}|}$$

$$R_{2} = I(X_{2}^{'}; X_{2}) = \frac{1}{2} \log \frac{|\mathbf{R}_{2}|}{|\mathbf{R}_{e'2}|}$$

$$R_{1} + R_{2} = I(X_{1}^{'}; X_{1}) + I(X_{2}^{'}; X_{2}) = \frac{1}{2} \log \frac{|\mathbf{R}_{x}|}{|\mathbf{R}_{e}|} = I(X;Y)$$

$$\left(\mathbf{R}_{x}^{-1} + \mathbf{H}^{T}\mathbf{H}\right)^{-1} = \begin{bmatrix}\mathbf{R}_{1}^{-1} + \mathbf{H}_{1}^{T}\mathbf{H}_{1} & \mathbf{H}_{1}^{T}\mathbf{H}_{2} \\ \mathbf{H}_{2}^{-1}\mathbf{H}_{1} & \mathbf{R}_{2}^{-1} + \mathbf{H}_{2}^{T}\mathbf{H}_{2}\end{bmatrix}^{-1} = \mathbf{G}^{-1}\Delta^{-1}\mathbf{G}^{-T}$$

$$G = \begin{bmatrix}\mathbf{I} & (\mathbf{R}_{x}^{-1} + \mathbf{H}_{1}^{T}\mathbf{H}_{1})^{-1} \mathbf{H}_{1}^{T}\mathbf{H}_{2} \\ \mathbf{0} & \mathbf{I}\end{bmatrix}^{-1}$$

$$\Delta^{-1} = \begin{bmatrix} (\mathbf{R}_{1}^{-1} + \mathbf{H}_{1}^{T}\mathbf{H}_{1})^{-1} & \mathbf{0} \\ (\mathbf{R}_{2}^{-1} + \mathbf{H}_{2}^{T}\mathbf{H}_{2} - \mathbf{H}_{2}^{T}\mathbf{H}_{1}(\mathbf{R}_{1}^{-1} + \mathbf{H}_{1}^{T}\mathbf{H}_{2})^{-1} \end{bmatrix}$$
Multiply APSA Mester Merit





$$R_{1} = I(X_{1}'; X_{1}) = \frac{1}{2} \log \frac{|\mathbf{R}_{1}|}{\left| \left(\mathbf{R}_{1}^{-1} + \mathbf{H}_{1}^{T} \mathbf{H}_{1} \right)^{-1} \right|} = \frac{1}{2} \log \left| \mathbf{H}_{1}^{T} \mathbf{R}_{1} \mathbf{H}_{1} + \mathbf{I} \right| = I(X_{1}; Y / X_{2})$$

$$R_{2} = I(X_{2}'; X_{2}) = \frac{1}{2} \log \frac{|\mathbf{R}_{2}|}{\left| \left(\mathbf{R}_{2}^{-1} + \mathbf{H}_{2}^{\mathsf{T}} \left(\mathbf{I} + \mathbf{H}_{1} \mathbf{R}_{1} \mathbf{H}_{1}^{\mathsf{T}} \right)^{-1} \mathbf{H}_{2} \right)^{-1} \right|} = \frac{1}{2} \log \frac{\left| \mathbf{H}_{1}^{\mathsf{T}} \mathbf{R}_{1} \mathbf{H}_{1} + \mathbf{H}_{2}^{\mathsf{T}} \mathbf{R}_{2} \mathbf{H}_{2} + \mathbf{I} \right|}{\left| \mathbf{H}_{1}^{\mathsf{T}} \mathbf{R}_{1} \mathbf{H}_{1} + \mathbf{I} \right|} = I(X_{2}; Y)$$

$$R_{1} + R_{2} = I(X_{1}, X_{2}; Y) = \frac{1}{2} \log \left| \mathbf{H}_{1}^{\mathrm{T}} \mathbf{R}_{1} \mathbf{H}_{1} + \mathbf{H}_{2}^{\mathrm{T}} \mathbf{R}_{2} \mathbf{H}_{2} + \mathbf{I} \right| = I(X_{2}; Y) + I(X_{1}; Y / X_{2})$$

Aside from the SVD decomposition, other matrix factorizations are going to be considered along this chapter : Cholesky factorization consists in where A is square and B is a lower triangular matrix; LU factorization consists in where A is square, L is lower triangular, D is diagonal and UH is upper triangular; QR factorization where A does not need to be square, Q is orthonormal and R is upper triangular.



Therefore, matrix B is completed with G-1 implemented in a feedback way, in order to preserve capacity as appendix B shows. Thanks to the feedback implementation, the precoder follows a Dirty Paper philosophy. For instance, the transmitter first picks a codeword for receiver 2 with full (noncausal) knowledge of the codeword intended for receiver 1. Therefore, receiver 2 does not see the codeword intended for receiver 1 as interference. Similarly, the codeword for receiver 3 is chosen such that receiver 3 does not see the signals intended for receivers 1 and 2 as interference. This process continues for all K receivers. Receiver 1 subsequently sees the signals intended for all other users as interference, receiver 2 sees the signals intended for users 3 to K as interference, etc. Note that the ordering of the users clearly matters in such a procedure.



The QR precoder

H = Q.R



with
$$B = Q^H$$

The MIMO chanel reduces to lower triangular. This is the link with the DP implementation (degraded channel)

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... because of downloads bottleneck

- Linear and non-linear precoding
- •ChannelState Inform ation feedback
- •Multiuser receivers
- •User selection and Scheduling strategies
- •Powercontroland other radio resource management

•D ifferent sights

- •From information theory point of view
- •From signalprocessing point of view
- •From network/protocolpointofview



SNIR – QoS vs MUD



Fairness: PFS QoS given Classical PHY approaches: - Minimum rate per user

- Average terms



More interesting for operator:

- Outage: to deliver service to the highest possible users and satisfy their minimum requirements

$$SNIR_{i,m} = \frac{1/n_t |h_i u_m|^2}{\sigma^2 + \sum_{u \neq m} \frac{1}{n_t} |h_i u_u|^2}$$

$$[F(x)]^{N} = 1 - \left[\frac{\exp(-xn_{t}\sigma^{2})}{(1+x)^{n_{t}-1}}\right]^{N}$$







Upper layers: Cross-layer design



Importance of throughput: PSR/sec









Example: The upper layer requirements can be in terms of Maximum packets delay





BC channel – Upper Layers



CAC: Call admission control



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BC channel – Upper Layers



Fairness

- Both the mean and the variance are shown
- Trade-off can be clearly identified
 - Global performance vs. individual needs
- Example
 - Number of antennas to attain a mean









Throughput-based









Distributed MIMO



Outline:

- Motivation
- Distributed MIMO scenarios
- Forwarding Strategies
 - Regenerative
 - Non-Regenerative
- D-MIMO in satellite communications







Motivation (1)



Distributed MIMO (virtual array): Obtain the MIMO gains using a collection of distributed antennas from multiple single antenna users.







Motivation (2) Benefits



MIMO gains:

- Diversity: Combat fading, from multi-path propagation.
- Beamforming Gains.
- Energy Savings, Capacity increases.
- Repeaters gains: Combat path loss and shadowing.
- Hardware gains: No need of co-located antenna elements.
- Others: (physical-layer) multi-hop routing.



Motivation (3) Drawbacks

- Share resources (can be difficult to justify,
- MIMO gains can't be fully obtained
 - e.g. 2x2 cooperative does not achieve multiplexing gain of 2.
- Distributed Channel Knowledge.
- Hardware limitations
 - Performance depends on the level synchronization between nodes: signal, symbol or non-

e.g1: Beamforming: needs signal synchronization

e.g2: Space-Time codes needs symbol synchronization

e.g3: If non-synchronization: Interference

Half-Duplex terminals

R1

R2

D



Distributed MIMO Scenarios (1)



MAC-based distributed MIMO

- Transmit diversity gain: distributed space-time codes or antenna selection.
- Capacity gain:
 - TX CSI: Optimum Beamforming.
 - No TX CSI: Multiplexing gain.





Distributed MIMO Scenarios (2)



BC-based distributed MIMO

- Receive diversity gain: antenna selection.
- Capacity gain:
 - With relays synchronization: Maximal ratio combining at the destination.
 - With no relays synchronization: Selection combining.
 - Requisites: good relay-destination channels.







Distributed MIMO Scenarios (3)



Point-to-point distributed MIMO

- Transmit/Receive diversity: space-time coding or antenna selection.
- Capacity gain: beamforming.
 - TX/RX CSI: optimum beamforming
 - ▲ No TX CSI: space-time coding.





Forwarding Strategies



The relay: The virtual MIMO is based on terminals that are able to forward to destination the information from the source.



m: message

f: relaying function

Two basic relaying techniques:

- Regenerative: Relays decode, re-encode and transmit.
- Non-Regenerative: Relays process the received signal but not decode.









- Decode and Forward: Decode the message and re-transmit.
- **Parity Forwarding:** Decode the message and tx the parity bits.
 - m|s: m message and s parity bits $x_r(m|s)$.
 - **•** Relay decode m but tx s. $x_r(s)$.
- Partial Decode and Forward: only a part of the message is relayed, the other part is directly transmitted to destination.
 - First Fase: Source $tx x(m_1)$ and relay decode.
 - Second Fase: Source tx $x(m_2)$ and Relay $tx x_r(m_1)$.

enefits:

The noise is completely removed at the relays.

rawbacks

The rate is limited by the decoding requirement at the relays. (channel between source and relay must be good).





Amplify and Forward: Relay transmits an amplified version of its received signal.

- ✓ f: x_r=a*y_r.
- Simple and cost-less implementation.
- Compress and Forward: Relay compresses its received signal with certain distortion, and transmits it to destination.
 - f: Wyner-Ziv compression $x_r = WZC(y_r)$.
 - Generally higher computational complexity than DF.

Benefits

- Good if relay is close to the destination node.
- Drawbacks
 - AF: Noise amplification at the relay.
 - CF: The rate is limited by the channel between relay and destination.