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A gateway to advanced communication technologies

SPACE-TIME CODING

Miguel Ángel Lagunas

m.a.lagunas@cttc.es

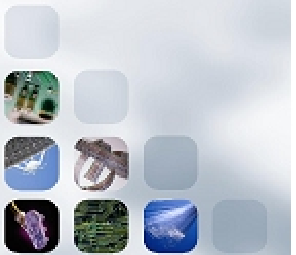
STBC Codes (space-time block)

For R input bits, there is an alphabet C which contains 2^R space time code-words size n_T by n_T

As stated before, any valid code-word verifies:

$$\underline{\underline{C}}_m \underline{\underline{C}}_m^H = \underline{\underline{I}}_{nT}$$

Let us assume that the transmitted code-word is C_1 and, assuming ML detection, we would like to compute the probability of error, i.e. decide C_0 when C_1 is the actual word



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It will be an error when:

$$Pe\left(\underline{\underline{C}}_0 \Rightarrow \underline{\underline{C}}_1\right) = \Pr\left(\left|\underline{\underline{X}}_{Rn} - \underline{\underline{H}}\,\underline{\underline{C}}_1\right|_F < \left|\underline{\underline{X}}_{Rn} - \underline{\underline{H}}\,\underline{\underline{C}}_0\right|_F\right)$$

Using that…

$$\underline{\underline{X}}_{Rn} = \sqrt{\frac{2E_s}{N_o}}\,\underline{\underline{H}}\,\underline{\underline{C}}_1 + \underline{\underline{W}}_n = \rho^{0.5}\,\underline{\underline{H}}\,\underline{\underline{C}}_1 + \underline{\underline{W}}_n$$

$$\left|\underline{\underline{X}}_{Rn} - \underline{\underline{H}}\,\underline{\underline{C}}_1\right|_F < \left|\underline{\underline{X}}_{Rn} - \underline{\underline{H}}\,\underline{\underline{C}}_0\right|_F$$

$$tr\left(\underline{\underline{X}}_{Rn}^H\,\underline{\underline{H}}\left(\underline{\underline{C}}_1 - \underline{\underline{C}}_0\right) + \left(\underline{\underline{C}}_1 - \underline{\underline{C}}_0\right)^H \underline{\underline{H}}^H\,\underline{\underline{X}}_{Rn}\right) < 0$$

$$tr\left(\underline{\underline{H}}\left(\underline{\underline{C}}_1 - \underline{\underline{C}}_0\right)\underline{\underline{C}}_1^H\,\underline{\underline{H}}^H + \underline{\underline{H}}\,\underline{\underline{C}}_1\left(\underline{\underline{C}}_1 - \underline{\underline{C}}_0\right)^H \underline{\underline{H}}^H\right) < \left(\underline{\underline{\tilde{C}}}\,\underline{\underline{H}}^H + \underline{\underline{H}}\,\underline{\underline{\tilde{C}}}\right)\underline{\underline{W}}_n$$

$$tr\left(\underline{\underline{H}}\left(2\underline{\underline{I}} - \underline{\underline{C}}_0\underline{\underline{C}}_1^H - \underline{\underline{C}}_1\underline{\underline{C}}_0^H\right)\underline{\underline{H}}^H\right) < tr\left(\left(\underline{\underline{\tilde{C}}}\,\underline{\underline{H}}^H + \underline{\underline{H}}\,\underline{\underline{\tilde{C}}}^H\right)\underline{\underline{W}}_n\right)$$

$$tr\left(\underline{\underline{H}}\left|\underline{\underline{\tilde{C}}}\right|_F\underline{\underline{H}}^H\right) < 2tr\left(\left(\underline{\underline{H}}\,\underline{\underline{\tilde{C}}}^H\right)\underline{\underline{W}}_n\right)$$

Error when a Gaussian distributed variable surpasses a value equal to its variance divided by 4.

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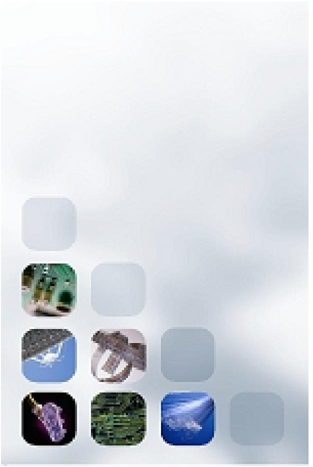

Since……

$$tr\left(\underline{\underline{H}}\left|\underline{\underline{\tilde{C}}}\right|_F \underline{\underline{H}}^H\right) = tr\left(\underline{\underline{H}}^H \underline{\underline{H}}\left|\underline{\underline{\tilde{C}}}\right|_F\right) = tr\left(\underline{\underline{H}}^H \underline{\underline{H}}\left(\underline{\underline{C}}_1 - \underline{\underline{C}}_0\right)^H\left(\underline{\underline{C}}_1 - \underline{\underline{C}}_0\right)\right) = tr\left(\underline{\underline{R}}_H \underline{\underline{A}}\right)$$

In consequence the probability of error will be:…..

$$P_e\left(\underline{\underline{C}}_0 \Rightarrow \underline{\underline{C}}_1\right) = Q\left(\sqrt{\frac{2E_s}{N_0} tr\left(\underline{\underline{R}}_H \underline{\underline{A}}\right)}\right)$$

For moderate and high SNR regimes…

$$P_e\left(\underline{\underline{C}}_0 \Rightarrow \underline{\underline{C}}_1\right) = Q\left(\sqrt{\frac{2E_s}{N_0} tr\left(\underline{\underline{R}}_H \underline{\underline{A}}\right)}\right) \approx kte \; \boxed{e^{-\frac{E_s}{N_0} tr\left(\underline{\underline{R}}_H \underline{\underline{A}}\right)}}$$

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The average probability of error can be obtained from the expected value of the previous one with respect the channel. Assuming the channel Gaussian distributed (No LOS condition), we have…..

$$P_e = E_H\left[\prod_{p=1}^{n_R} e^{-\frac{E_s}{N_0}\underline{h}_p^H \underline{\underline{A}}\,\underline{h}_p}\right] = \prod_{p=1}^{n_R}\int e^{-\frac{E_s}{N_0}\underline{h}_p^H \underline{\underline{A}}\,\underline{h}_p}\,\frac{e^{-\underline{h}_p^H \underline{\underline{\Sigma}}_p^{-1}\underline{h}_p}}{\det\left(\underline{\underline{\Sigma}}_p\right)}\,d\underline{h}_p$$

Where vector h_p denotes the channel viewed from antenna p at the receiver, i.e. channel from the n_T antennas a_t Tx to antenna p at Rx. It is also assumed that antennas are well separated and, in consequence, the n_R vectors are independent each other.

Solving the integral……. $P_e = \prod_{p=1}^{n_R} \frac{1}{\det\left(\underline{\underline{I}}_{nT} + \frac{2E_s}{N_0} \underline{\underline{\Sigma}}_p \underline{\underline{A}} \right)}$

In addition assuming that all the Rx antennas experience the same variance h_0^2 at moderate and high SNR regimes we can use that

$$P_e \propto \left(\frac{2E_s h_0^2}{N_0} \right)^{-n_T n_R} \left(\det^{1/n_T} \left(\underline{\underline{A}} \right) \right)^{-n_T n_R}$$

Thus, the code gain is:

$$\boxed{\det^{1/n_T} \left(\underline{\underline{A}} \right)}$$

OSTBC Codes (Multiplexing)

The coder matrix $$\underline{\underline{B}}.\underline{\underline{B}}^H = \underline{\underline{I}}_{n_T}$$

The Tx and Rs signals as well as the estimated symbol is:

$$\underline{\underline{X}}_{T,n} = \underline{\underline{B}}.s1(n)$$

$$\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{X}}_{T,n} + \underline{\underline{W}}_n$$

$$\widehat{s1} = Traza\left[\underline{\underline{B}}^H \underline{\underline{H}}^H .\underline{\underline{X}}_{R,n}\right]$$

For nt=2 there are several possibilities with entries entailing no operation

$$\begin{pmatrix}1&0\\0&1\end{pmatrix}\quad\begin{pmatrix}-1&0\\ & \end{pmatrix}\quad\begin{pmatrix}-1&0\\ & \end{pmatrix}\quad\begin{pmatrix}1&0\\ &-1\end{pmatrix}$$

$$\begin{pmatrix}0&1\\1&0\end{pmatrix}\quad\cdots\quad\begin{pmatrix} &1\\1&0\end{pmatrix}\quad\ldots$$

Gain 3 dB as code it is more a DSP than a code.????

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For two PAM symbols (real) and 2 antennas

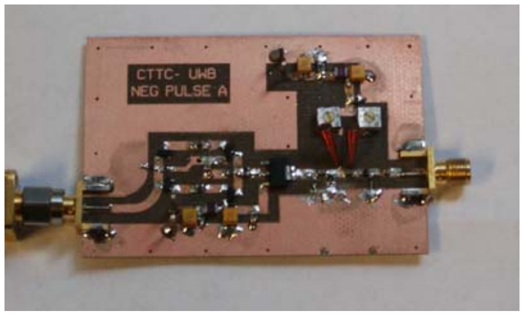
$$\underline{\underline{X}}_T = \underline{\underline{B}}_1 . s1 + \underline{\underline{B}}_2 . s2$$

$$\hat{s}1 = \underbrace{Traza\left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_H . \underline{\underline{B}}_1\right]}_{desired} . s1 + \underbrace{Traza\left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_H . \underline{\underline{B}}_2\right]}_{ISI} . s2$$

The no-ISI constrain is:

$$Traza\left[\underline{\underline{B}}_1^H . \underline{\underline{R}}_H . \underline{\underline{B}}_2\right] = Traza\left[\underline{\underline{R}}_H . \underline{\underline{B}}_2 . \underline{\underline{B}}_1^H\right] = 0 \quad \Rightarrow \quad \underline{\underline{B}}_2 . \underline{\underline{B}}_1^H = \underline{\underline{0}}\ ?$$

Not orthogonal just to be amicable



$$\underline{\underline{B}}_1 . \underline{\underline{B}}_2^H = -\underline{\underline{B}}_2 . \underline{\underline{B}}_1^H$$

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$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1 .s1 + \underline{\underline{B}}_2 .s2 = \begin{pmatrix} s1 & -s2 \\ s2 & s1 \end{pmatrix}$$

To further achieve full-rate, we need two additional matrices, that being amicable in order to detect two imaginary parts, do not promote ISI with the real symbols.

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \underline{\underline{B}}_3 \quad \underline{\underline{B}}_4$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1 .s1 + \underline{\underline{B}}_2 .s2 + j.\underline{\underline{B}}_3 .s3 + j.\underline{\underline{B}}_4 .s4$$

It is easy to check that
this is the constraint---→

$$\underline{\underline{B}}_1 .\underline{\underline{B}}_3^H = \underline{\underline{B}}_3 .\underline{\underline{B}}_1^H$$

$$\underline{\underline{B}}_2 .\underline{\underline{B}}_4^H = \underline{\underline{B}}_4 .\underline{\underline{B}}_2^H$$

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In summary:

$$\underline{\underline{B}}_1.\underline{\underline{B}}_1^H = \underline{\underline{B}}_2.\underline{\underline{B}}_2^H = \underline{\underline{B}}_3.\underline{\underline{B}}_3^H = \underline{\underline{B}}_4.\underline{\underline{B}}_4^H = \underline{\underline{I}}_2$$

$$\underline{\underline{B}}_1.\underline{\underline{B}}_2^H = -\underline{\underline{B}}_2.\underline{\underline{B}}_1^H \qquad \underline{\underline{B}}_4.\underline{\underline{B}}_3^H = -\underline{\underline{B}}_3.\underline{\underline{B}}_4^H$$

$$\underline{\underline{B}}_1.\underline{\underline{B}}_3^H = \underline{\underline{B}}_3.\underline{\underline{B}}_1^H \qquad \underline{\underline{B}}_2.\underline{\underline{B}}_4^H = \underline{\underline{B}}_4.\underline{\underline{B}}_2^H$$

The Alamouti's code:

$$\underline{\underline{B}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \underline{\underline{B}}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \underline{\underline{B}}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \underline{\underline{B}}_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{\underline{X}}_T = \underline{\underline{B}}_1.s1 + \underline{\underline{B}}_2.s2 + \underline{\underline{B}}_3.j.s3 + \underline{\underline{B}}_4.j.s4 = \begin{pmatrix} s1 + j.s3 & -s2 + js4 \\ s2 + j.s4 & s1 - j.s3 \end{pmatrix} = \begin{pmatrix} z1 & -z2^* \\ z2 & z1^* \end{pmatrix}$$

The receiver------>

$$\hat{s}1 = \mathrm{Re}\left(Traza\left[\underline{\underline{B}}_1^H.\underline{\underline{R}}_H.\underline{\underline{X}}_R\right]\right)$$

$$\hat{s}2 = \mathrm{Re}\left(Traza\left[\underline{\underline{B}}_2^H.\underline{\underline{R}}_H.\underline{\underline{X}}_R\right]\right)$$

$$\hat{s}3 = \mathrm{Im}\left(Traza\left[\underline{\underline{B}}_1^H.\underline{\underline{R}}_H.\underline{\underline{X}}_R\right]\right)$$

$$\hat{s}4 = \mathrm{Im}\left(Traza\left[\underline{\underline{B}}_1^H.\underline{\underline{R}}_H.\underline{\underline{X}}_R\right]\right)$$

Unfortunately no such full-rate codes exist for any number of antennas. There are solution for rates lower than one like the code shown below for 4 antennas and rate ¾.

$$\begin{bmatrix} s1 & 0 & s2 & -s3 \\ 0 & s1 & s3^* & s2^* \\ -s2^* & -s3 & s1^* & 0 \\ s3^* & -s2 & 0 & s1^* \end{bmatrix}$$

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Convolutional S-T Codes: Trellis codes

output input state

$$\underline{x} = \left\lfloor \underline{\underline{G}}_1 . \underline{a} + \underline{\underline{G}}_2 . \underline{b} \right\rfloor^{\square}$$

Measurement equation

$$\underline{b} = \left\lfloor \underline{\underline{G}}_3 . \underline{a} + \underline{\underline{G}}_4 . \underline{b} \right\rfloor^{\oplus}$$

State equation

$\underline{a}=[a(1),a(2),....,a(R)]^T$ input bits

L bits per component output $\underline{x}$

L=2 bits Components 0(00),1(01),2(10),3(11) that correspond to the four signals in a QPSK constellation

Code rate=R/L over n_T antennas

K bits for M components of the state vector $\underline{b}$

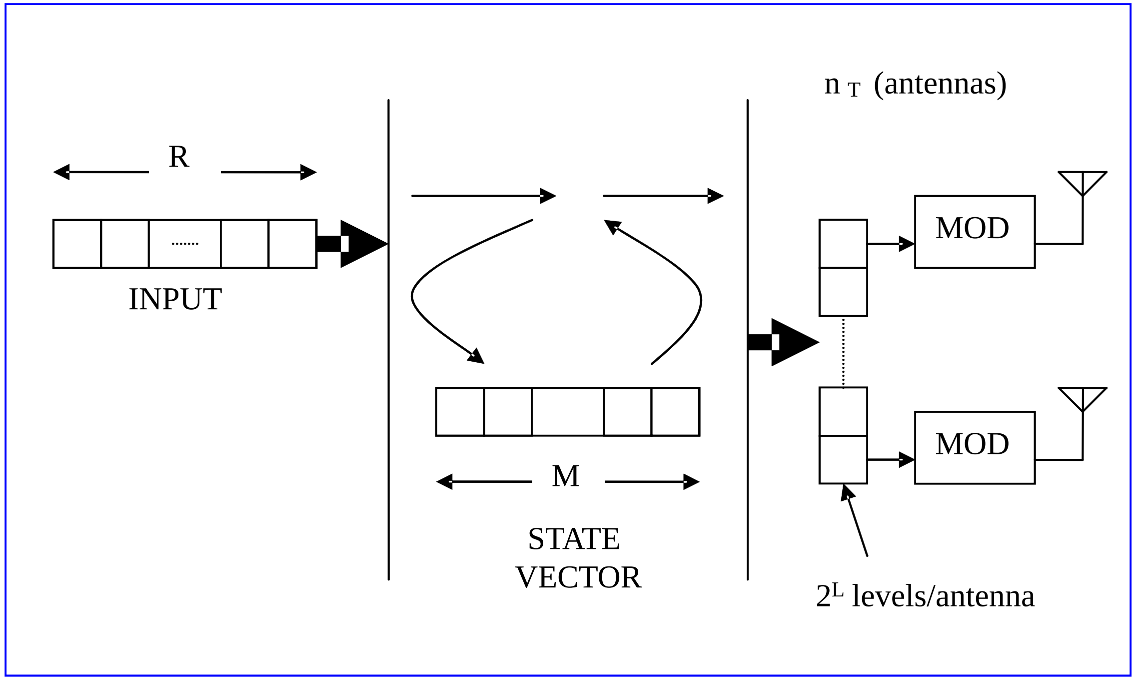
Code specification:

-Number of Tx antennas n_T

-Bits/Hz or size of the radiated constelation L

-Complexity at Tx or number of states $2^{K.M}$

-Code Rate equal to R/L



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A more compact formulation of the state model is:

Grouping input and state vectors in a single one

$$\underline{c} = \begin{bmatrix} a(1) & \ldots & a(R) & b(1) & \ldots & b(M) \end{bmatrix}$$

$$\underline{x} = \left\lfloor \underline{\underline{G}}.\underline{c} \right\rfloor^{\oplus}$$

And, the state vector is formed by the last M bits after a shift of a given number of components vector $\underline{c}$ from left to right

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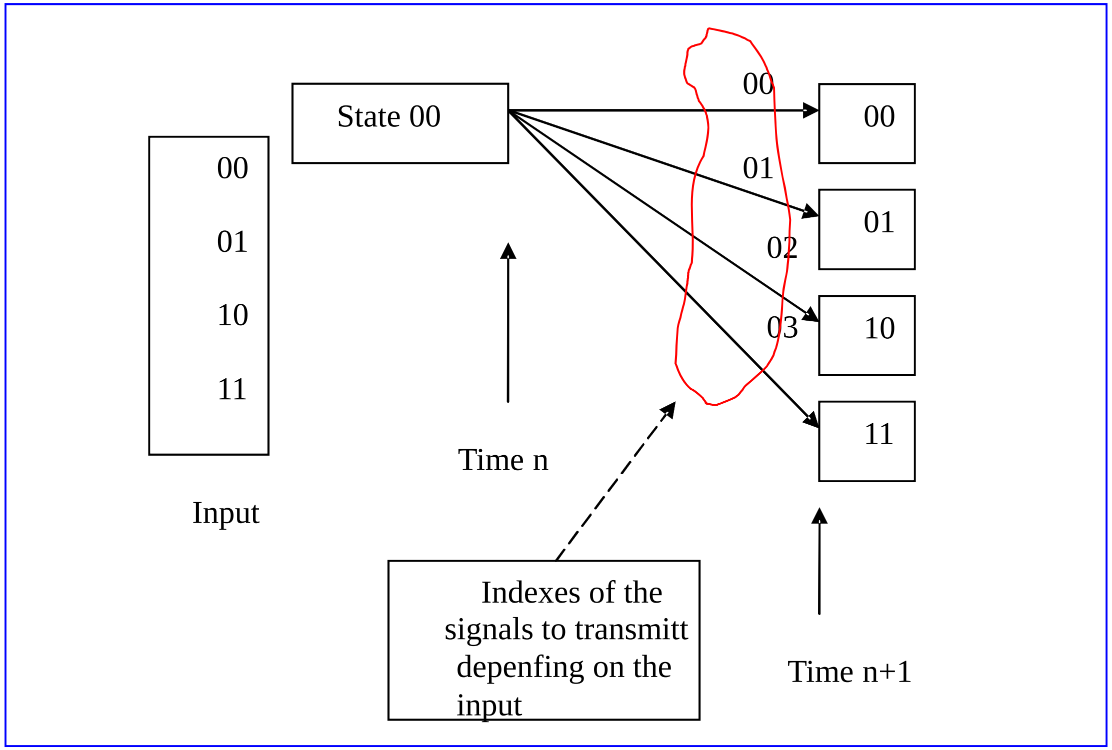
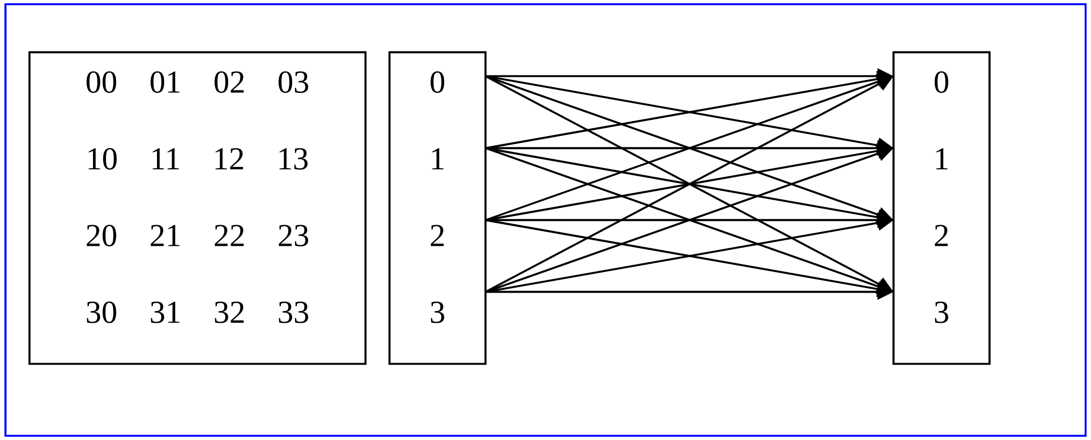
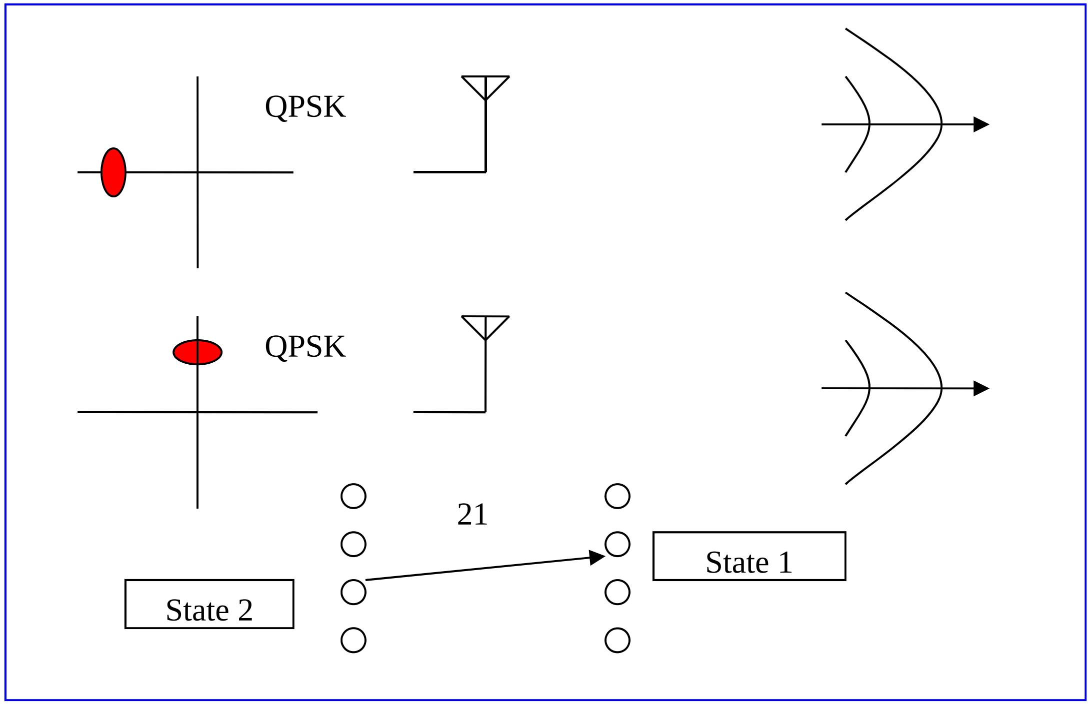
Code st2bh2est4rate1: (binary codes)

$$\begin{bmatrix} a(1) & a(2) & b(1) & b(2) \end{bmatrix} . \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 2 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x1 & x2 \end{bmatrix}$$

Modul 2 operation for x1 and x2




00 01 02 03
10 11 12 13
20 21 22 23
30 31 32 33
0
1
2
3
0
1
2
3


QPSK
QPSK
21
State 2
State 1

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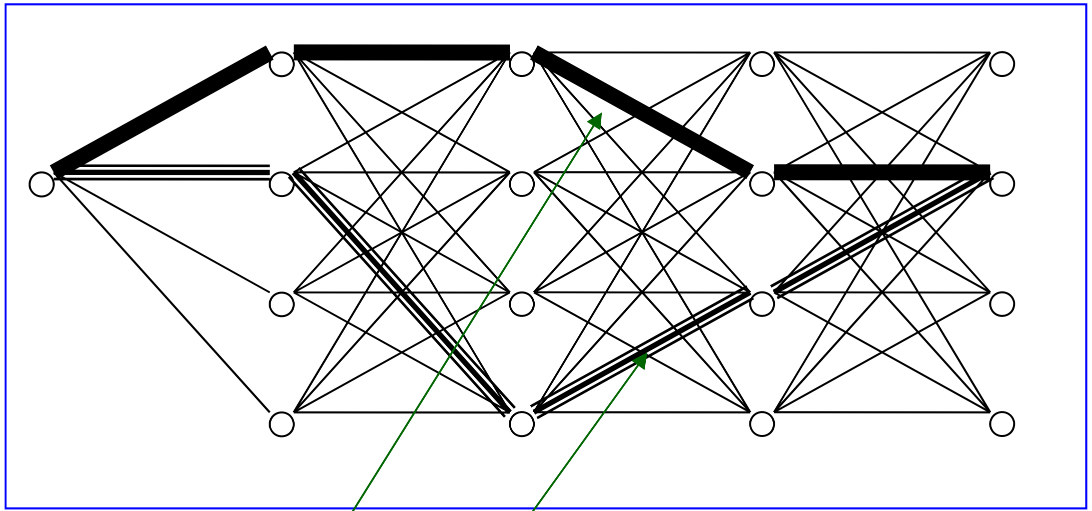

For uniform code any two pair is good. Use the (0,0,0,0,0…) as reference.

Search for the lowest distance covered to recover the zero path (This will be the worst matrix A to be used in the BER upper bound

$$\Pr\left(\underline{s}_n \Rightarrow \underline{b}_n; n=1,N\right) = Q\left(\sqrt{\left(\frac{E_s}{2.N_0}\right).Traza\left(\underline{\underline{R}}_H . \underline{\underline{A}}\right)}\right)$$

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For a length of N channels access

$$\underline{\underline{A}} = \sum_{n=1}^{N} \left(\underline{s}_n - \underline{b}_n\right).\left(\underline{s}_n - \underline{b}_n\right)^H$$

And the average BER

$$P_e = E_H\left[\prod_{p=1}^{n_R} e^{-\frac{E_s}{N_0}\underline{h}_p^H \underline{\underline{A}}\underline{h}_p}\right] = \prod_{p=1}^{n_R}\int e^{-\frac{E_s}{N_0}\underline{h}_p^H \underline{\underline{A}}\underline{h}_p} \frac{e^{-\underline{h}_p^H \underline{\underline{\Sigma}}_p^{-1}\underline{h}_p}}{\det\left(\underline{\underline{\Sigma}}_p\right)} d\underline{h}_p$$

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) \approx k_1.\prod_{p=1}^{n_R} \frac{1}{\det\left[\underline{\underline{I}}_{n_T} + \frac{2E_S}{N_0}.\underline{\underline{A}}\underline{\underline{\Sigma}}_p\right]} \approx$$

For moderate and high SNRs

$$\approx k_1.\left(\frac{2E_S}{N_0}\right)^{-n_R.n_T}.\prod_{p=1}^{n_R}\frac{1}{\det\left[.\underline{\underline{A}}\underline{\underline{\Sigma}}_p\right]}$$

or

$$\Pr\left(\underline{I}_0 \Rightarrow \underline{I}_e\right) \approx k_1 . \left(\frac{2E_S}{N_0}\right)^{-n_R . n_T} . \prod_{p=1}^{n_R} \frac{1}{\det\left[\underline{\underline{A}} . \underline{\underline{\Sigma}}_p\right]} \approx k_2 . \left(\frac{2E_S}{N_0}\right)^{-n_R . n_T} . \left(\det\left(\underline{\underline{A}}\right)\right)^{-n_R}$$

$$\text{Code Gain} = \left[\det\left(\underline{\underline{A}}\right)^{1/n_T}\right]$$

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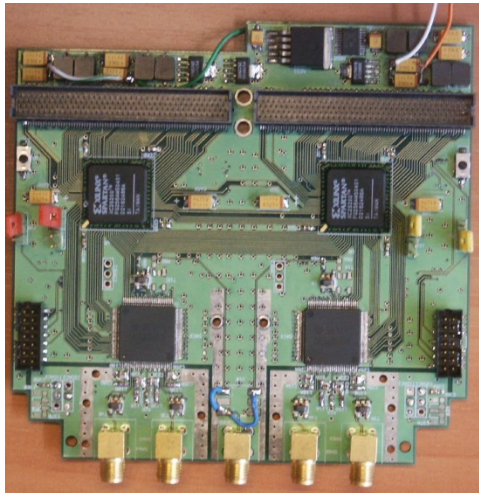

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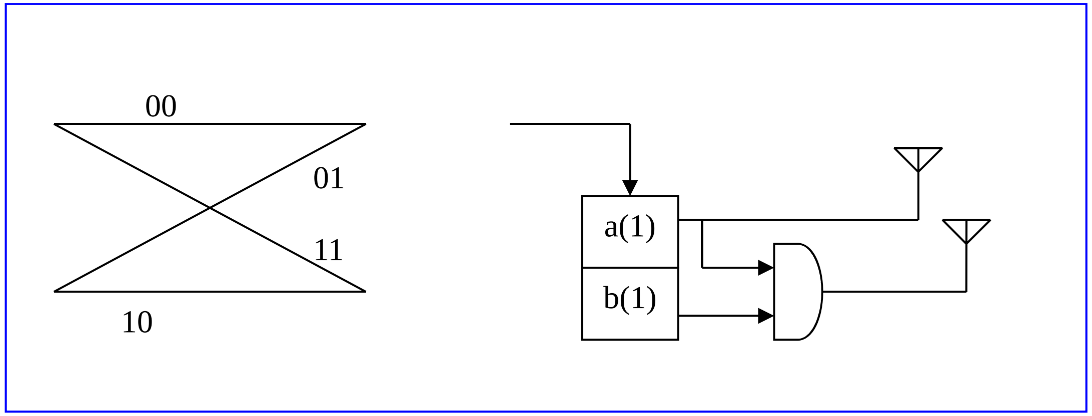
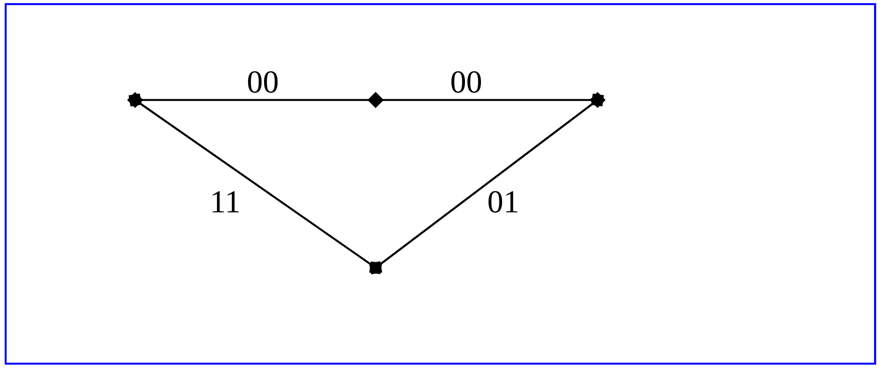
For st2bh1est2rate0.5

$$\underline{\underline{G}} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



$$\underline{s}_1 - \underline{b}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad y \quad \underline{s}_2 - \underline{b}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$



$$\underline{\underline{A}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.\begin{bmatrix} 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}.\begin{bmatrix} 0 & 2 \end{bmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 8 \end{pmatrix}$$

$$\det\left(\underline{\underline{A}}\right) = 16 \qquad Ganancia = \sqrt{16} = 4$$

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St2bh2est8 Gain $\sqrt{22}$ $$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 2 \end{bmatrix}$$

St2bh2est16 Gain $\sqrt{32}$ $$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 1 & 2 & 0 \\ 2 & 2 & 1 & 2 & 0 & 2 \end{bmatrix}$$

St2bh1est2 Gain 4 $$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

St2bh1est4 Gain $\sqrt{48}$ $$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 2 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 2 \end{bmatrix}$$

St2bh1est8 Gain $\sqrt{80}$ $$\underline{\underline{G}}^T = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

St2bh1est16 Gain $\sqrt{128}$ $$\underline{\underline{G}}^T = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

St3bh1est8 Gain $\sqrt[3]{256}$ $$\underline{\underline{G}}^T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

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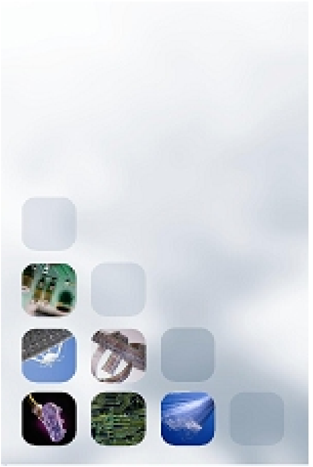

Codes for No-CSI at Rx

Assume that $\log_2 M$ bits have to be transmitted using N access to the channel with n_T antennas. We will use M matrixes of n_T by N

$$\underline{\underline{C}}_m \qquad m = 1, M$$

$$\underline{\underline{C}}_m . \underline{\underline{C}}_m^H = \underline{\underline{I}}_{n_T} \qquad \forall m = 1, M$$

UPA at Tx

H matrix is random-> Average BER

At Rx→ $\underline{\underline{Y}}_R = \left(\frac{2E_S}{N_0} \right)^{1/2} . \underline{\underline{H}} . \underline{\underline{C}}_0 + \underline{\underline{W}} = \rho^{1/2} . \underline{\underline{H}} . \underline{\underline{C}}_0 + \underline{\underline{W}}$

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$$\underline{\underline{\Sigma}} = E\left[\underline{\underline{Y}}_R^H . \underline{\underline{Y}}_R\right] = \underline{\underline{I}} + \underline{\underline{C}}_0^H . E\left[\underline{\underline{H}}^H . \underline{\underline{H}}\right] . \underline{\underline{C}}_0 = \underline{\underline{I}} + \rho . \underline{\underline{C}}_0^H . \underline{\underline{R}}_{HA} . \underline{\underline{C}}_0 = \underline{\underline{I}} + \rho \left|H_0\right|^2 . \underline{\underline{C}}_0^H . \underline{\underline{C}}_0$$

The ML receiver is:

$$\Pr\left(\underline{\underline{Y}}_R / \underline{\underline{C}}_0\right) = k_0 . \exp - \left[Traza\left(\underline{\underline{Y}} . \underline{\underline{\Sigma}}^{-1} . \underline{\underline{Y}}^H\right)\right] =$$

$$= k_0 . \exp - \left[Traza\left(\underline{\underline{Y}} . \left(\underline{\underline{I}} - \rho' . \underline{\underline{C}}_0^H . \underline{\underline{C}}_0\right) . \underline{\underline{Y}}^H\right)\right]$$

$$\hat{m} = \underset{\underline{\underline{C}}_m, m=1,M}{\arg\max} \left[Traza\left(\underline{\underline{Y}}_R . \underline{\underline{C}}_m^H . \underline{\underline{C}}_m . \underline{\underline{Y}}_R^H\right)\right]$$

Perfect detection occurs when:

$$Traza\left(\underline{\underline{Y}}_R . \underline{\underline{C}}_0^H . \underline{\underline{C}}_0 . \underline{\underline{Y}}_R^H\right) > Traza\left(\underline{\underline{Y}}_R . \underline{\underline{C}}_1^H . \underline{\underline{C}}_1 . \underline{\underline{Y}}_R^H\right)$$

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$$\rho.Traza\left[\underline{\underline{H}}.\left(\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H.\underline{\underline{C}}_1.\underline{\underline{C}}_0^H\right).\underline{\underline{H}}^H\right]>2\rho^{1/2}.\mathrm{Re}\left\{Traza\left[\underline{\underline{W}}.\left(\underline{\underline{C}}_0^H.\underline{\underline{C}}_0-\underline{\underline{C}}_1^H.\underline{\underline{C}}_1\right).\underline{\underline{C}}_0^H.\underline{\underline{H}}^H\right]\right\}$$

$$\Pr\left(\underline{\underline{C}}_0\to\underline{\underline{C}}_1\right)\approx Q\left(\sqrt{\left(\frac{2E_s}{N_0}\right).Traza\left(\underline{\underline{H}}.\underline{\underline{A}}_{NOCSI}.\underline{\underline{H}}^H\right)}\right)\approx$$

$$\approx k_1.\exp\left[-\left(\left(\frac{2E_s}{N_0}\right).Traza\left(\underline{\underline{H}}.\underline{\underline{A}}_{NOCSI}.\underline{\underline{H}}^H\right)\right)\right]$$

$$\Pr^{AVE}\approx k_2.\prod_{p=1}^{n_R}\frac{1}{\det\left[\underline{\underline{I}}-\left(\frac{E_s}{2.N_0}\right)\underline{\underline{A}}_{NOCSI}.\underline{\underline{\Sigma}}_p\right]}$$

$$E_S=\left(\frac{N}{n_T}\right).E_T$$

It is important to remark the loss due to the absence of CSI at Rx

$$\boxed{\begin{aligned}\underline{\underline{A}}_{CSI}&=\left(\underline{\underline{C}}_0-\underline{\underline{C}}_1\right)\left(\underline{\underline{C}}_0-\underline{\underline{C}}_1\right)^H\\ \underline{\underline{A}}_{NOCSI}&=\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H.\underline{\underline{C}}_1.\underline{\underline{C}}_0^H\end{aligned}}$$

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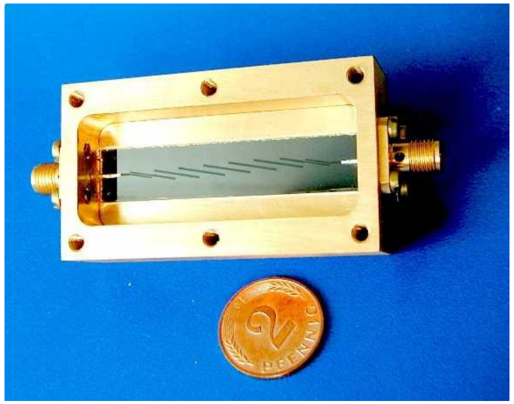
In order to compare both cases, note that:

$$\left|\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H\right|^2=\underline{\underline{I}}+\underline{\underline{C}}_0.\underline{\underline{C}}_1^H\,\underline{\underline{C}}_1.\underline{\underline{C}}_0^H-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H-\underline{\underline{C}}_1.\underline{\underline{C}}_0^H=$$

$$2.\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H-\underline{\underline{C}}_1.\underline{\underline{C}}_0^H-\underline{\underline{A}}_{NOCSI}=\underline{\underline{A}}_{CSI}-\underline{\underline{A}}_{NOCSI}$$

$$Asi \quad pues,$$

$$\underline{\underline{A}}_{CSI}=\underline{\underline{A}}_{NOCSI}+\left|\underline{\underline{I}}-\underline{\underline{C}}_0.\underline{\underline{C}}_1^H\right|^2$$



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Codigos ST Diferenciales

Assuming that there is CSi at Rx

$$\left|\underline{\underline{X}}_R - E_s^{1/2}.\underline{\underline{H}}.\underline{\underline{C}}_m\right|_F \Rightarrow \quad \hat{\underline{\underline{C}}} = \underset{\underline{\underline{C}}_m;m=1,M}{Max}\left[\mathrm{Re}\left(Traza\left(\underline{\underline{H}}.\underline{\underline{C}}_m.\underline{\underline{X}}_R\right)\right)\right]$$

The probability of error is:

$$Pe = \Pr\left(\underline{\underline{C}}_m \to \underline{\underline{C}}_n ; \underline{\underline{\tilde{C}}} \equiv \underline{\underline{C}}_m - \underline{\underline{C}}_n\right) = k_0.Q\left(\sqrt{\frac{E_s}{2.N_0}.Traza\left(\underline{\underline{R}}_H.\underline{\underline{\tilde{C}}}.\underline{\underline{\tilde{C}}}^H\right)}\right)$$

$$Pe \approx k_1.\exp\left[-\frac{E_s}{4.N_0}.Traza\left(\underline{\underline{R}}_H.\underline{\underline{\tilde{C}}}.\underline{\underline{\tilde{C}}}^H\right)\right] = k_1.\exp\left[-\frac{E_s}{4.N_0}.\sum_{p=1}^{n_R}\underline{h}_p^H.\underline{\underline{\tilde{C}}}.\underline{\underline{\tilde{C}}}^H.\underline{h}_p\right]$$

$$Pe^{aver} = k_2.\prod_{p=1}^{n_R}\frac{1}{\det\left[\underline{\underline{I}} + \frac{E_s}{4.N_0}.\underline{\underline{\tilde{C}}}.\underline{\underline{\tilde{C}}}^H.\underline{\underline{\Sigma}}_j\right]}$$

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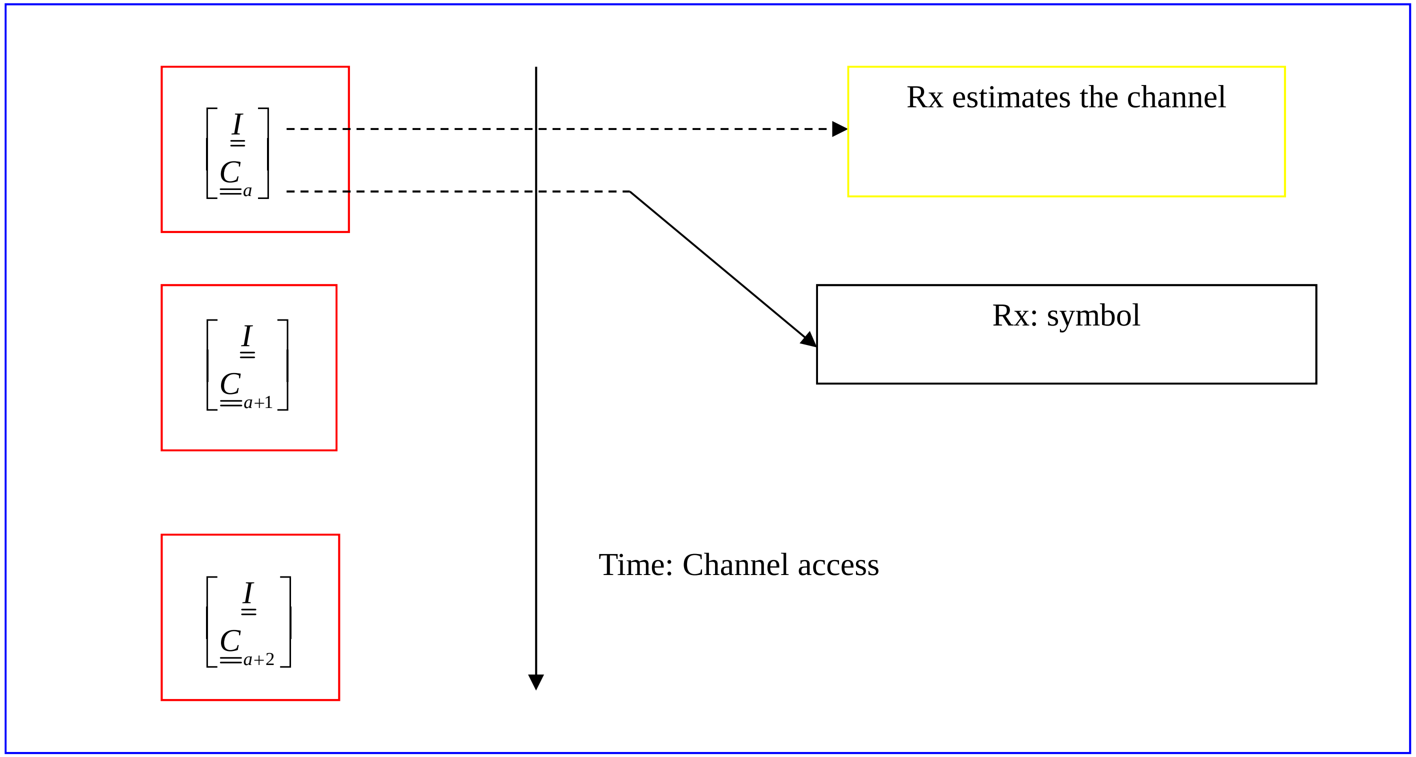
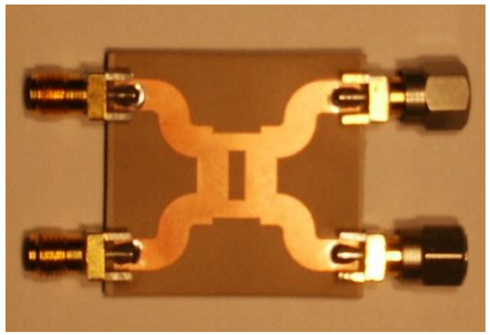
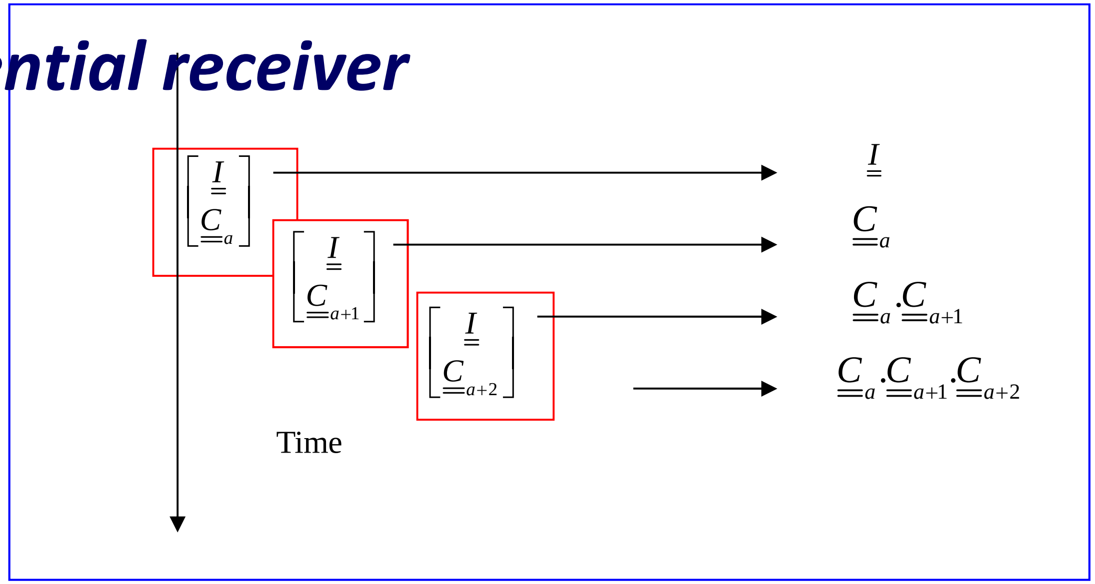
Using two symbols first estimate the channel second to decode.



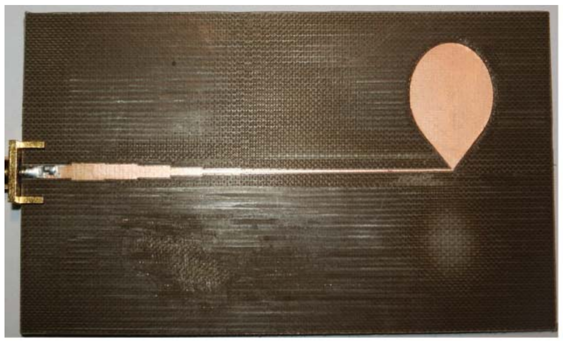


The differential receiver



The desired word received is $\underline{\underline{Z}}_{n-1} = \prod_{a=1} \underline{\underline{C}}_{n-a}$

And the received snapshot $\underline{\underline{X}}_{R,n-1} = \underline{\underline{H}} . \underline{\underline{Z}}_{n-1} + \underline{\underline{W}}_{n-1}$



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Hereafter it is shown that the ST code produces, in fact, a new channel together with 3 dB. increase of noise.

$$\underline{\underline{X}}_{R,n} = \underline{\underline{H}}.\underline{\underline{Z}}_{n-1}.\underline{\underline{C}}_{n} + \underline{\underline{W}}_{n} = \left(\underline{\underline{X}}_{R,n-1} - \underline{\underline{W}}_{n-1}\right).\underline{\underline{C}}_{n} + \underline{\underline{W}}_{n}$$

$$\underline{\underline{X}}_{R,n} = \underline{\underline{X}}_{R,n-1}.\underline{\underline{C}}_{n} + \left(\underline{\underline{W}}_{n} - \underline{\underline{W}}_{n-1}.\underline{\underline{C}}_{n}\right) = \underline{\underline{H}}_{nuevo}.\underline{\underline{C}}_{n} + \underline{\underline{W}}_{nuevo}$$

Regardless the system is full-rate the decoder requires of two received codewords

$$\left|\underline{\underline{X}}_{R,n} - \underline{\underline{X}}_{R,n-1}.\underline{\underline{C}}_{n}\right|_{F} \Rightarrow \quad \underline{\underline{\widehat{C}}} = \underset{\underline{\underline{C}}_{n};n=1,M}{Max}\left[\mathrm{Re}\left(Traza\left(\underline{\underline{X}}_{R,n-1}.\underline{\underline{C}}_{n}.\underline{\underline{X}}^{H}_{R,n}\right)\right)\right]$$

Also, the matrix A for the average BER is as follows:

$$2\,\underline{\underline{I}} - \left(\underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_0.\underline{\underline{C}}_1^H.\underline{\underline{Z}}_{k-1}^H + \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_1.\underline{\underline{C}}_0^H.\underline{\underline{Z}}_{k-1}^H\right) =$$

$$= \cdot\left[\left(\underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_0 - \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_1\right)\cdot\left(\underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_0 - \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_1\right)^H\right] =$$

$$= \boxed{\cdot\left[\underline{\underline{Z}}_{k-1}\cdot\left(\underline{\underline{C}}_0 - \underline{\underline{C}}_1\right)\cdot\left(\underline{\underline{C}}_0 - \underline{\underline{C}}_1\right)^H .\underline{\underline{Z}}_{k-1}^H\right] = \underline{\underline{A}}_{DIF}}$$

Where, taking into account the orthogonal character of the received code-words and the commutative property of the determinant, results identical to the CSI at Rx case with 3dB loss.

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Some examples of differential ST codes

For 1 bit rate only two matrixes $\Phi = \pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

With initial matrix $\underline{\underline{D}} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

BPSK as constellation and rate 0.25 and 2 antennas, Gain 4

2 bits, 2 antennas, rate 0.5→ four matrixes, BPSK, Gain 4

$$\Phi = \left\{ \pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \pm\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\}$$

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QPSK, 2bits/seg/Hz, 8 codewords, Code gain4, Rate 1

$$\Phi = \left\{ \pm\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \pm\begin{pmatrix} j & 0 \\ 0 & -j \end{pmatrix}, \pm\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \pm\begin{pmatrix} 0 & j \\ j & 0 \end{pmatrix} \right\}$$

"quaternion" similar to Alamouti's code

For higher rates the codewords are formed as:

$$w_Q = \exp\left(j2\pi / Q \right)$$

$$\Phi = \left\{ \begin{pmatrix} 0 & w_Q \\ 1 & 0 \end{pmatrix}^m ; m = 0, \ldots Q-1 \right\}$$

Q=8 Rate 2 Gain 1.531 // Q=16 Rate 2.5 Gain 0.7804

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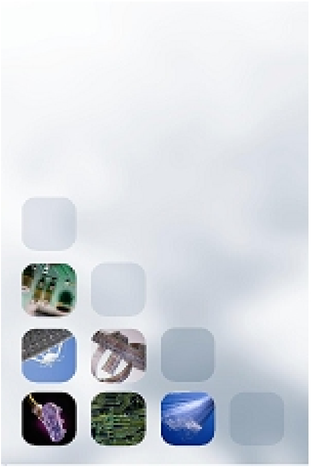

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Parc Mediterrani de la Tecnologia (PMT)

Av. Canal Olímpic S/N 08860 – Castelldefels Barcelona (Spain)

Voice: +34 93 645 29 00 :: Fax: +34 93 645 29 01

E-mail: info@cttc.es :: Web: http://www.cttc.es

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A compact formulation for two channel access:

Two data received $\left[\underline{\underline{X}}_{R,k-1} \quad \underline{\underline{X}}_{R,k}\right]$

New codeword $\left[\underline{\underline{Z}}_{k-1} \quad \underline{\underline{Z}}_{k-1}.\underline{\underline{C}}_{k}\right] \equiv \overline{\underline{\underline{C}}}_{k}$

with

$$\overline{\underline{\underline{C}}}_{k}^{H}.\overline{\underline{\underline{C}}}_{k} = \begin{pmatrix} \underline{\underline{I}} & \underline{\underline{C}}_{k} \\ \underline{\underline{C}}_{k}^{H} & \underline{\underline{I}} \end{pmatrix} \quad y \quad \overline{\underline{\underline{C}}}_{k}.\overline{\underline{\underline{C}}}_{k}^{H} = 2.\underline{\underline{I}}$$

Optimum detector for no-CSI at Rx that arrives to the same result

$$Traza\left[\left(\underline{\underline{X}}_{R,k-1} \quad \underline{\underline{X}}_{R,k}\right)\begin{pmatrix} \underline{\underline{I}} & \underline{\underline{C}}_{k} \\ \underline{\underline{C}}_{k}^{H} & \underline{\underline{I}} \end{pmatrix}\begin{pmatrix} \underline{\underline{X}}_{R,k-1}^{H} \\ \underline{\underline{X}}_{R,k} \end{pmatrix}\right] = Traza\left[\underline{\underline{X}}_{R,k}\underline{\underline{C}}_{k}^{H}\underline{\underline{X}}_{R,k-1}^{H}\right]$$